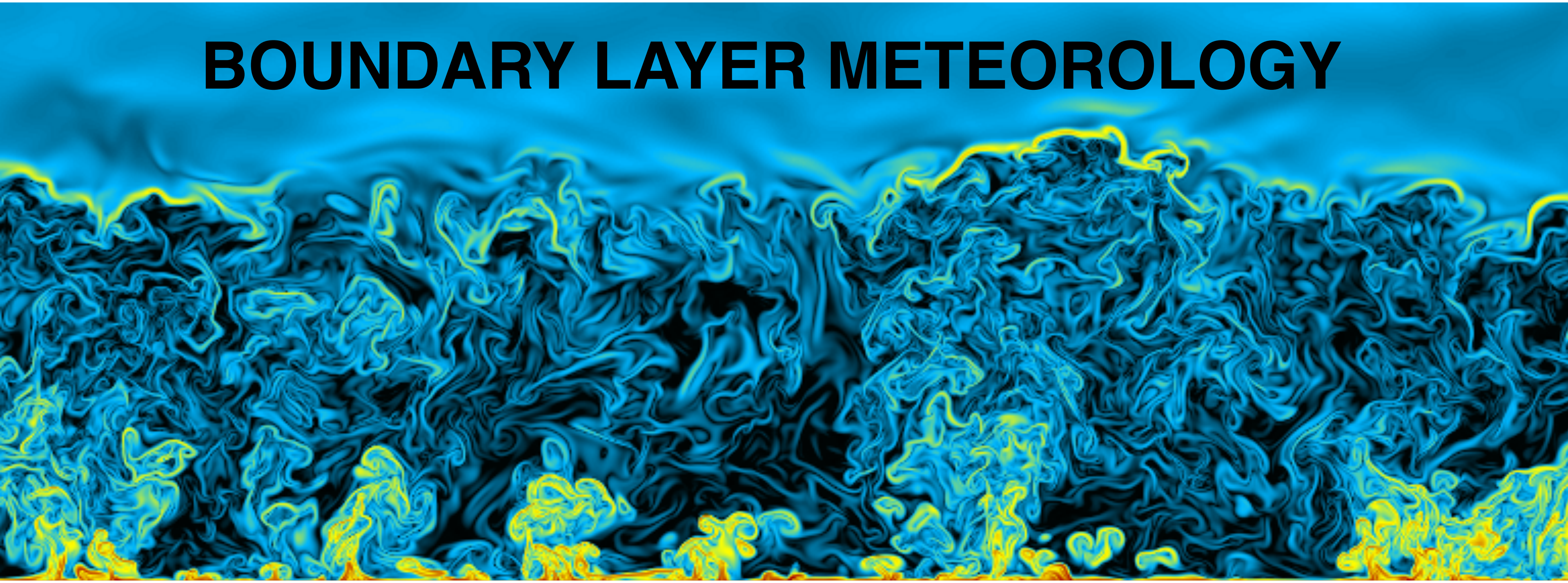
BOUNDARY LAYER METEOROLOGY



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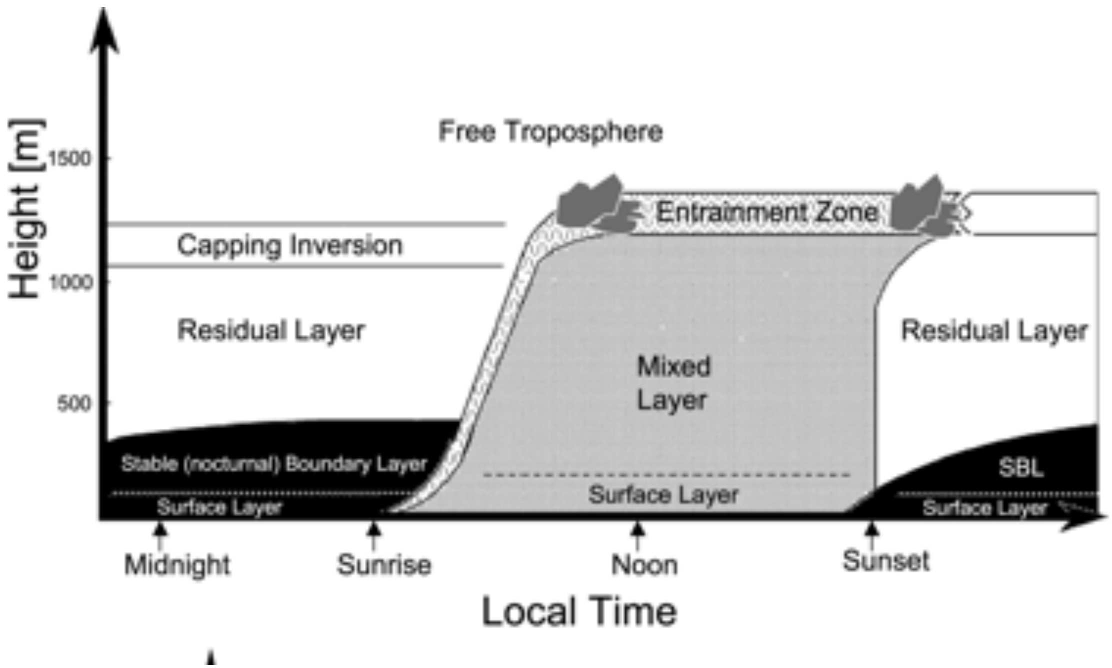
Department of Atmospheric and Cryospheric Sciences

Chapter 4

Similarity Theory

Similarity Theory

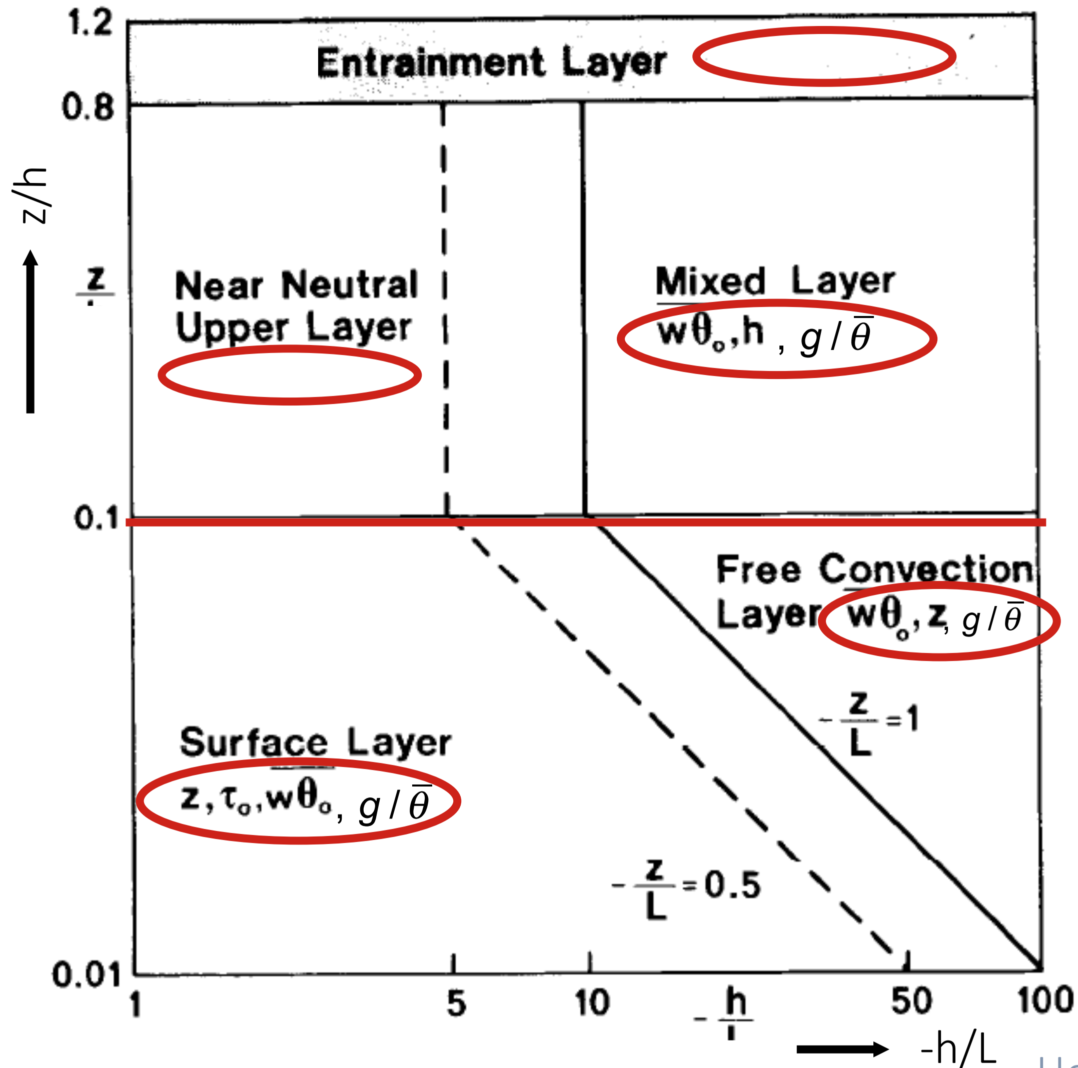
→ one example shown: MOST (Surface Layer)
→ in the boundary layer: different *Scaling Regimes* exist



→ Boundary Layer in 'phase space'
→ horizontally homogeneous (presumed)
→ non-dimensional height vs. non-dimensional stability

Holtslag and Nieuwstadt (1986)

Scaling regimes: Unstable

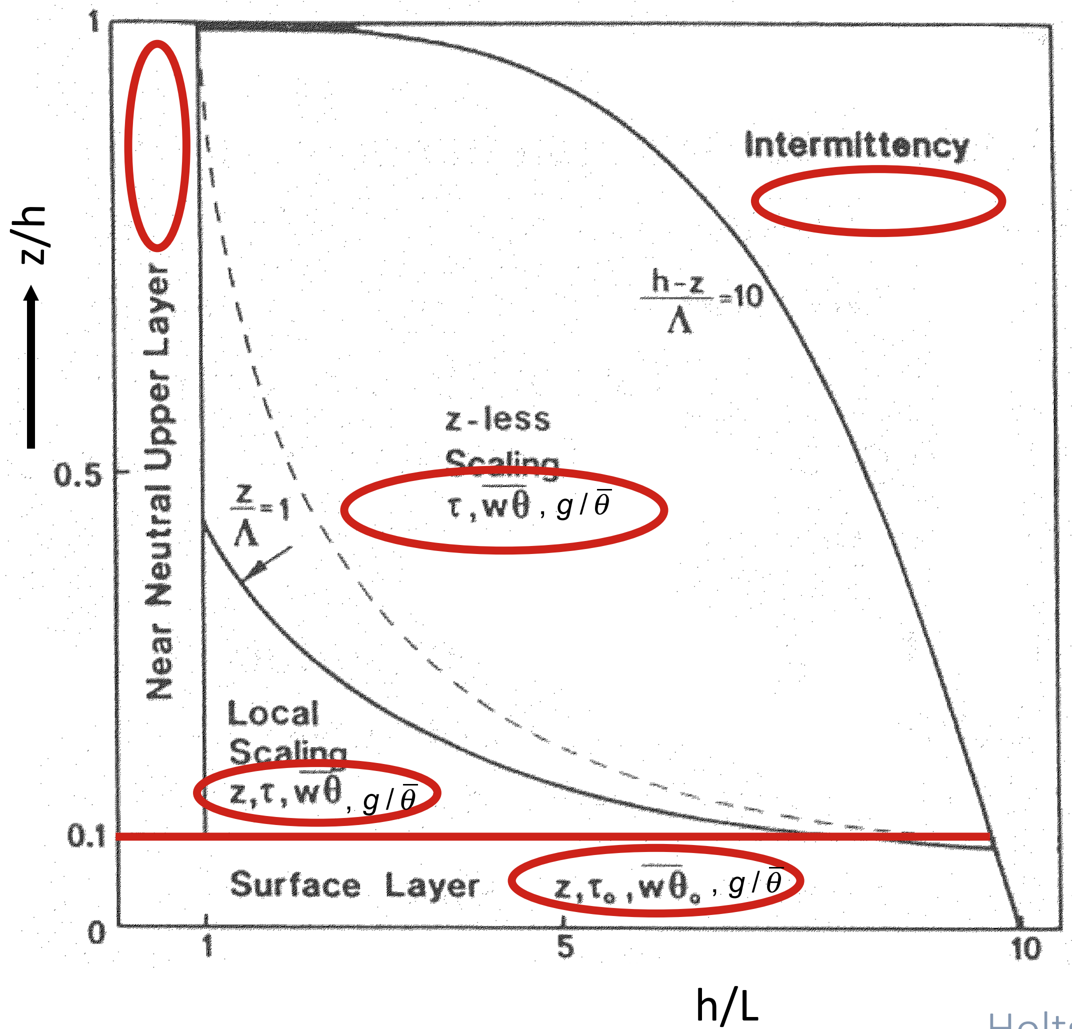


$$h \triangleq z_i$$

$$L \equiv -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'_o}}\left(\frac{g}{\overline{\theta}}\right)^{-1}$$

Holtslag and Nieuwstadt (1986)

Scaling regimes: Stable



$$h \triangleq z_i$$

$$L \equiv -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'}_o}\left(\frac{g}{\bar{\theta}}\right)^{-1}$$

Holtslag and Nieuwstadt (1986)

Scaling regimes

→ 'always' Surface Layer (MOST) at the bottom
→ regimes *with* variables: 'successful'
→ regimes without: 'not successful'

'successful' :
→ T_f = 'forcing time scale'
→ T_m= Time scale 'mean profile'

→ $T_f >> T_m$: quasi stationary

quasi stationary ↔ successful

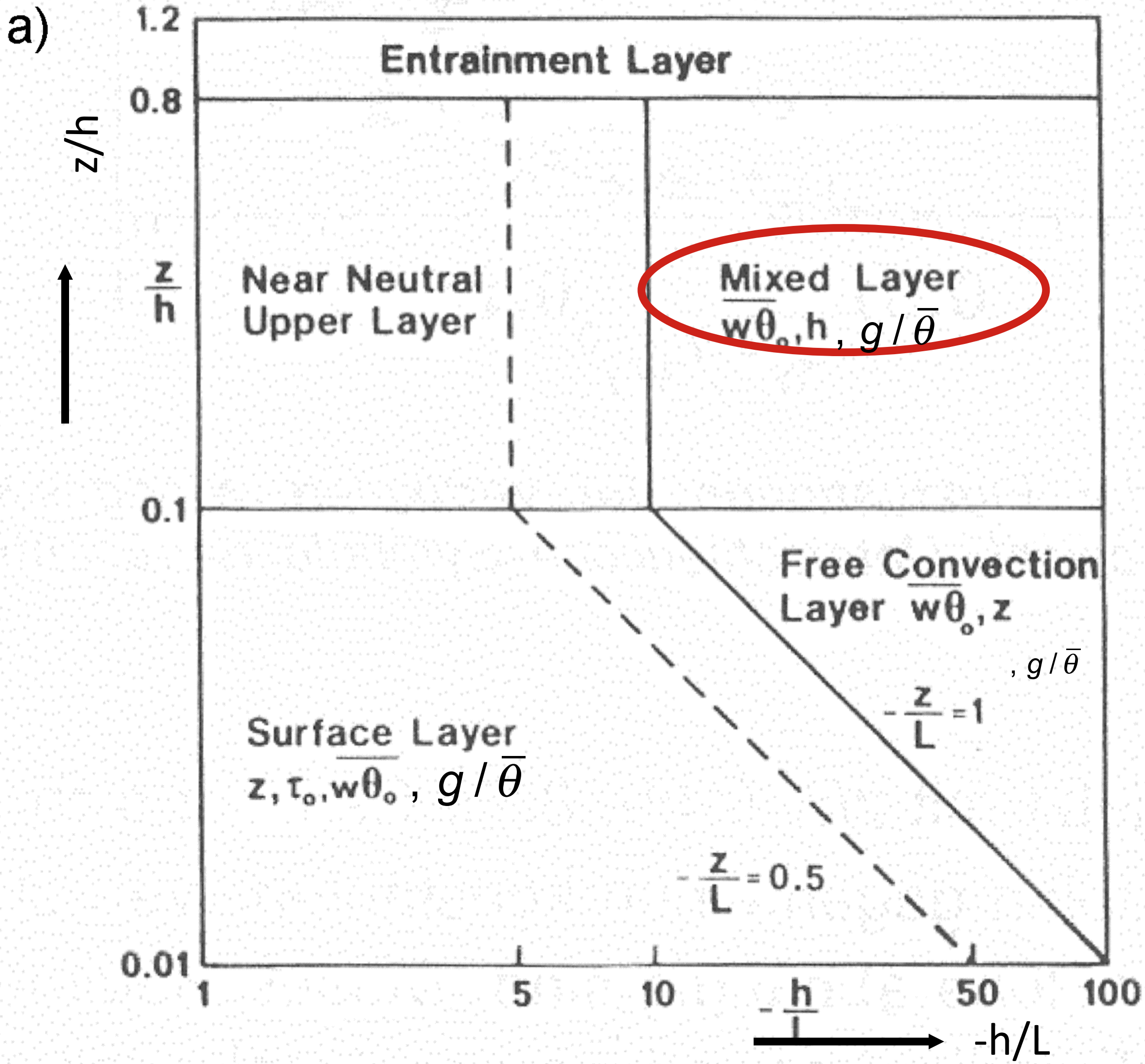
Scaling regimes: The successful Three

The successful three

→ Surface Layer (MOST) ✓
(including Free Convection limit)

→ Mixed Layer

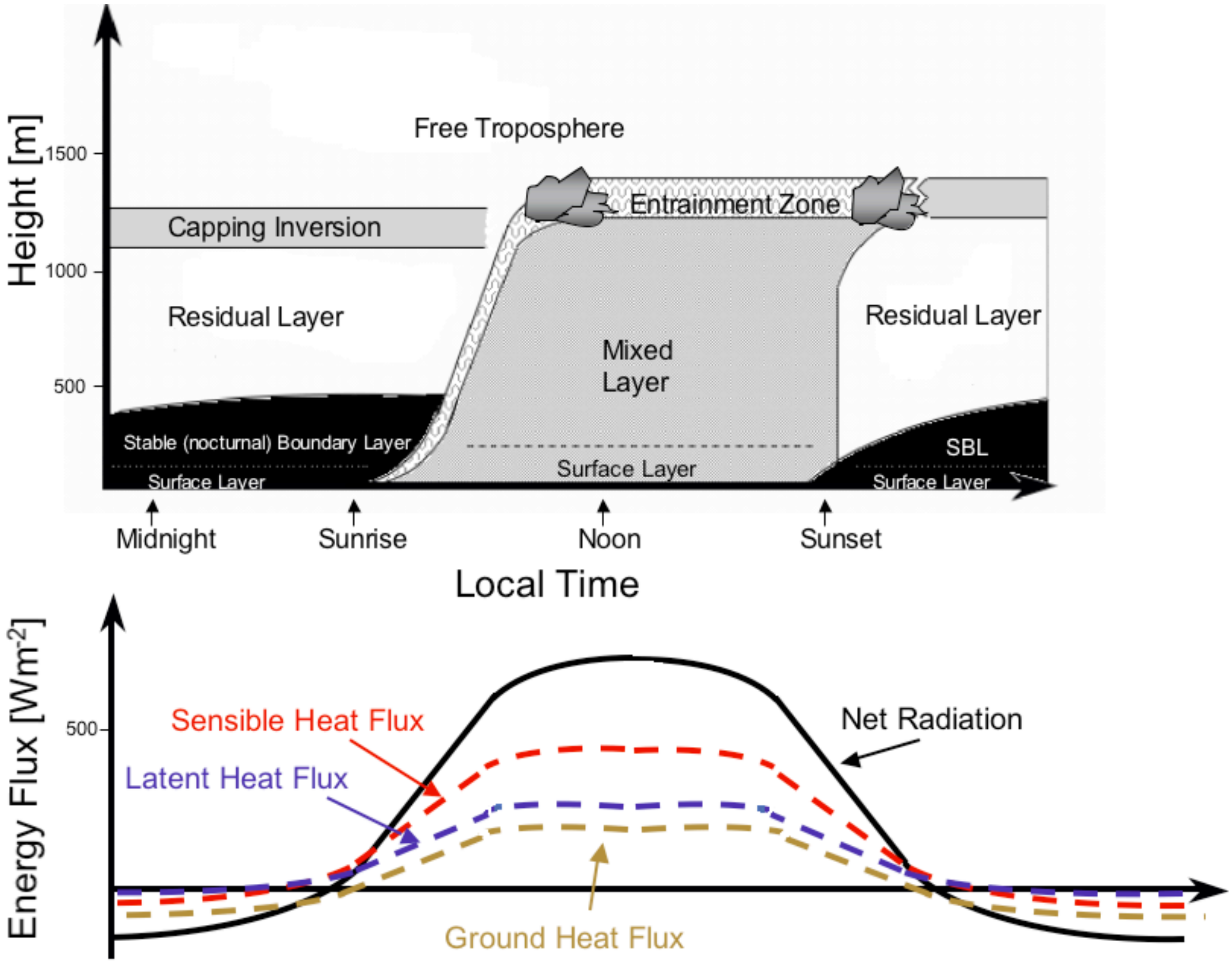
Scaling regimes: Mixed Layer



$$h \triangleq z_i$$

$$L \equiv -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'_o}}\left(\frac{g}{\bar{\theta}}\right)^{-1}$$

Scaling regimes: Mixed Layer



Scaling regimes: Mixed Layer

Processes:

$\overline{w'\theta'}_o$	heating at the ground	
z_i	entrainment	N=1
$g/\overline{\theta}$	buoyancy	$\pi_1 = z/z_i$
z	length scale	

$$\frac{\overline{a}}{a_*} = f_a\left(\frac{z}{z_i}\right)$$

Scaling regimes: Mixed Layer

velocity scale:

→ dimensional analysis:

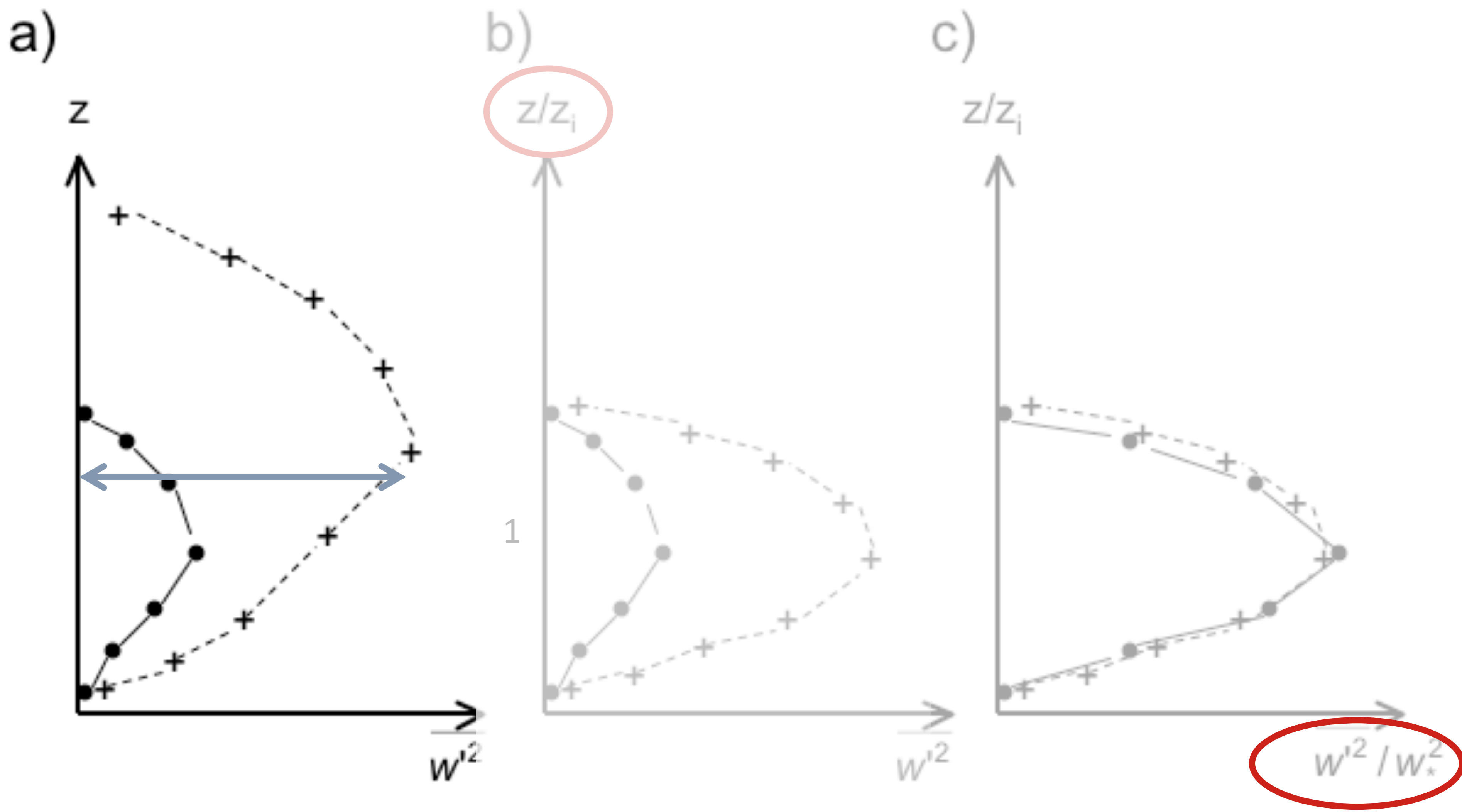
$$w_* = \left(\frac{g}{\overline{\theta}}\overline{w'\theta'}_o z_i\right)^{1/3}$$

temperature scale:

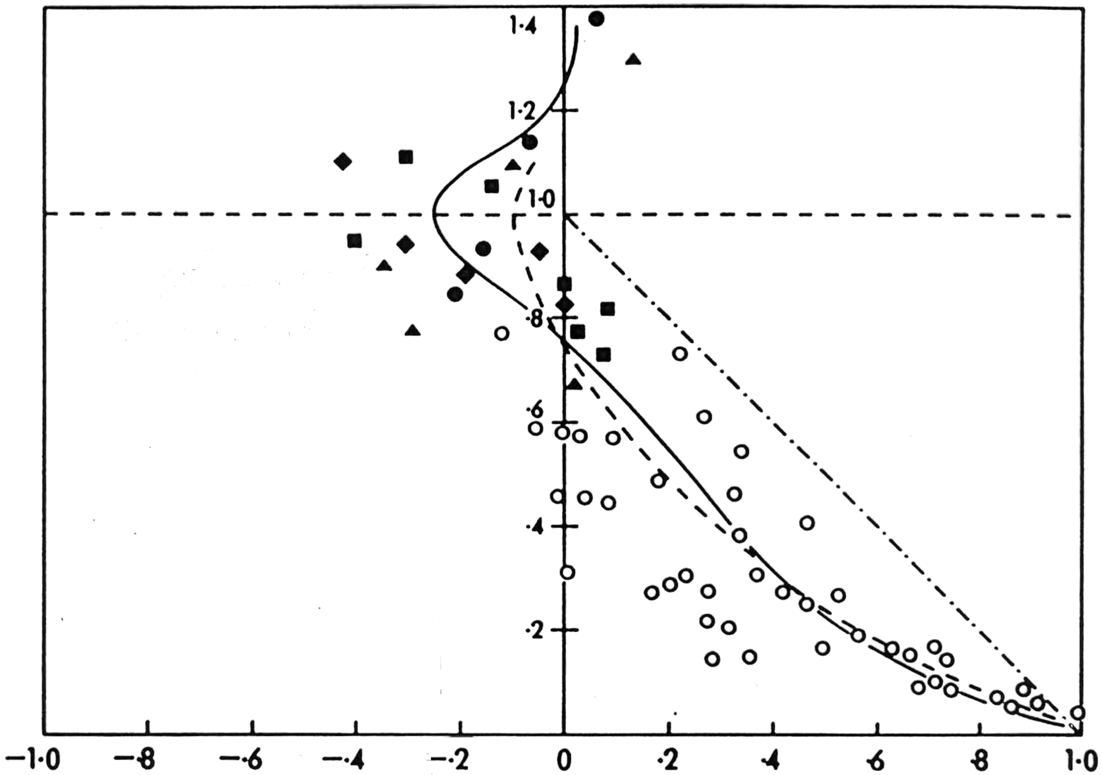
→ dimensional analysis:

$$\theta_{*CBL} = \overline{w'\theta'}_o / w_*$$

Scaling regimes: Mixed Layer



Scaling regimes: Mixed Layer

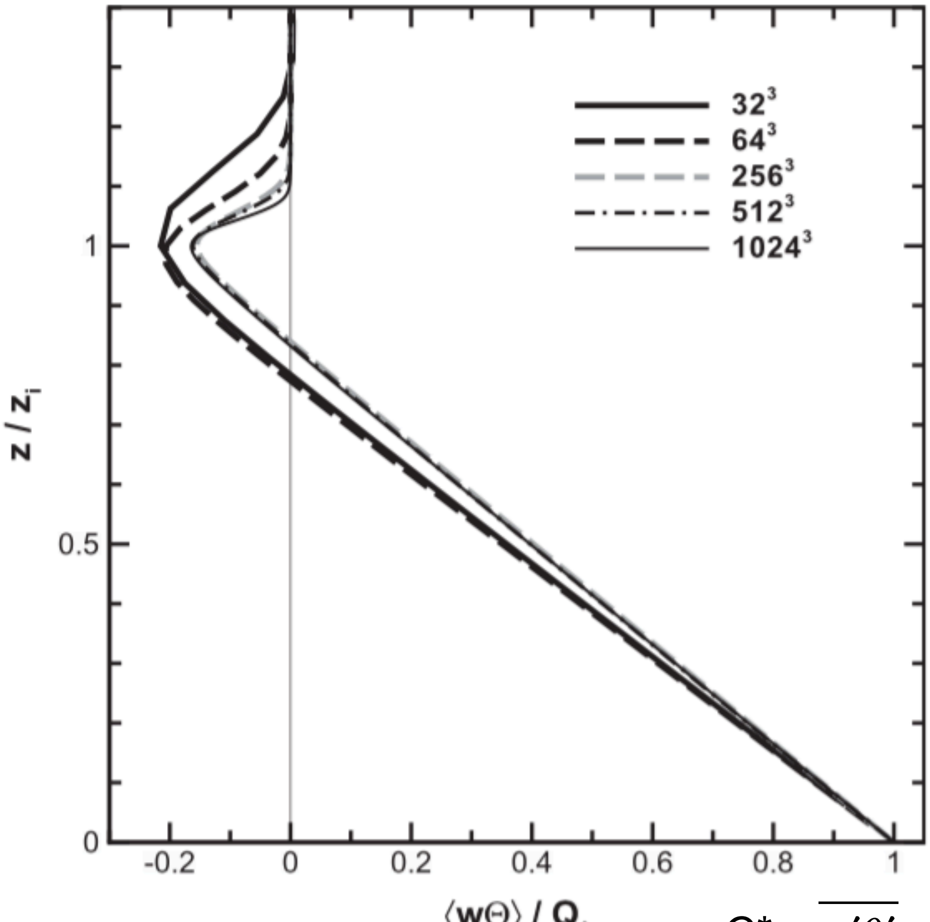


$\overline{w'\theta'} / \overline{w'\theta'_o}$

CBL
→ different measurements

symbols:
different flight campaigns
→ difficult measure- ments
→ 'large' scatter

Scaling regimes: Mixed Layer



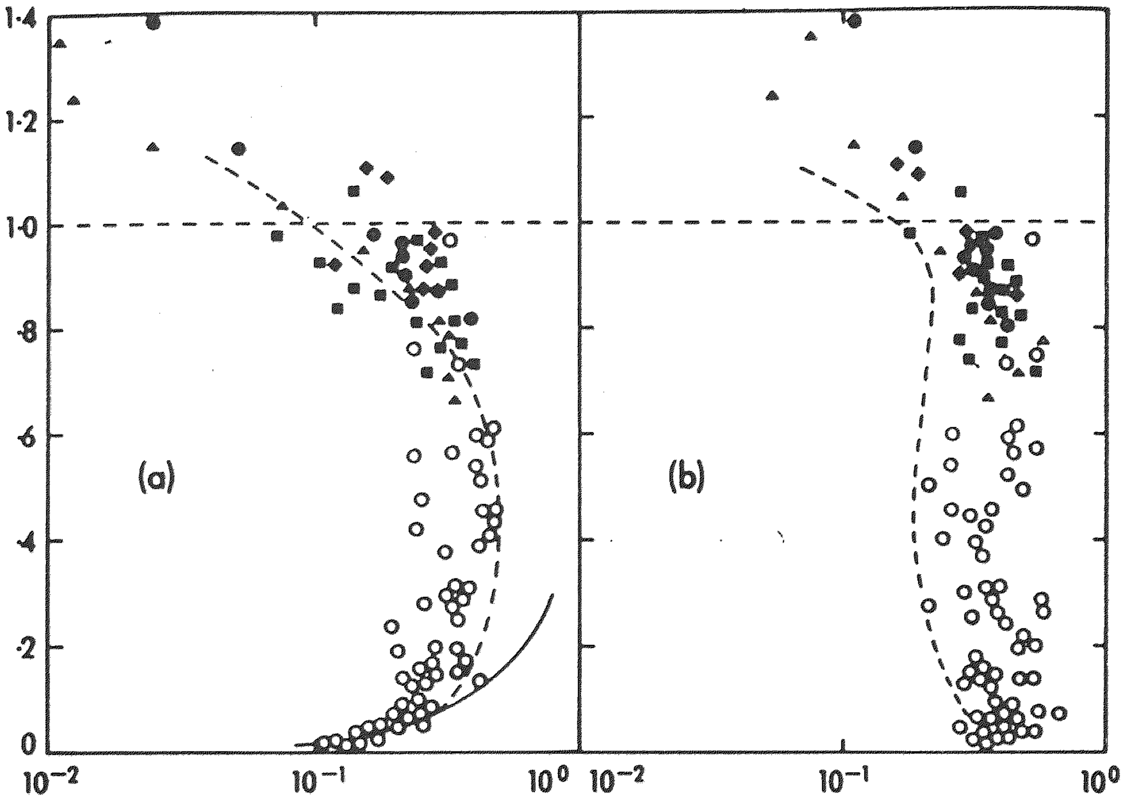
$Q^* = \overline{w'\theta'_o}$

LES of CBL
→ different resolutions
→ 'no' scatter under ideal conditions

Sullivan and Patton (2011)
DOI: 10.1175/JAS-D-10-05010.1

Scaling regimes: Mixed Layer

z/z_i



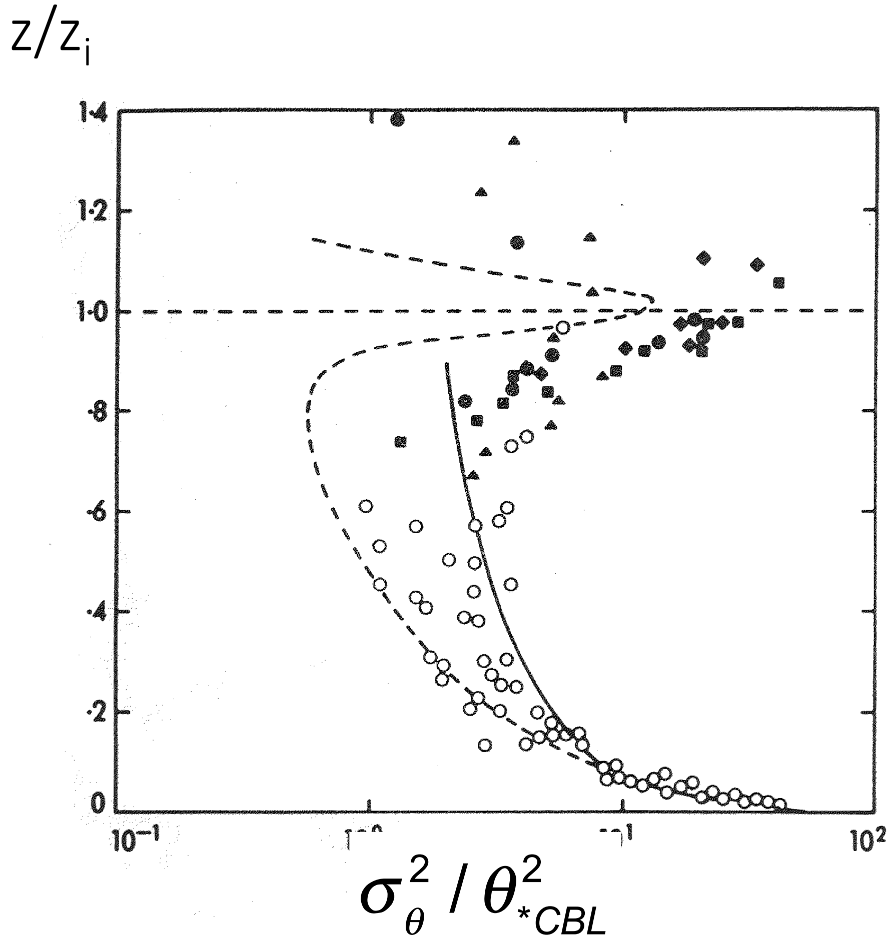
σ_w^2 / w_*^2 $\sigma_{u,v}^2 / w_*^2$

CBL

→ different measurements (symbols)

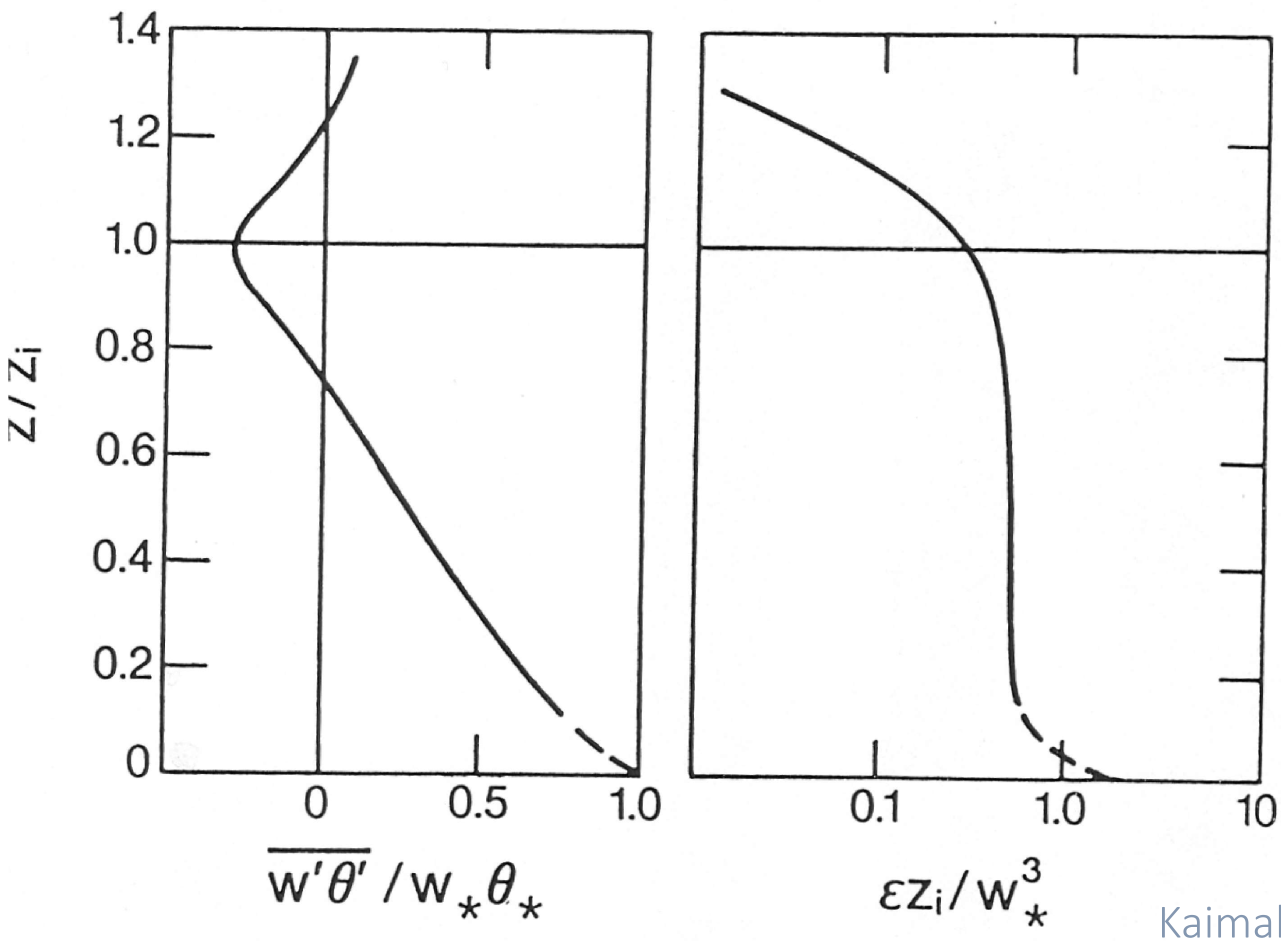
universität innsbruck

Scaling regimes: Mixed Layer



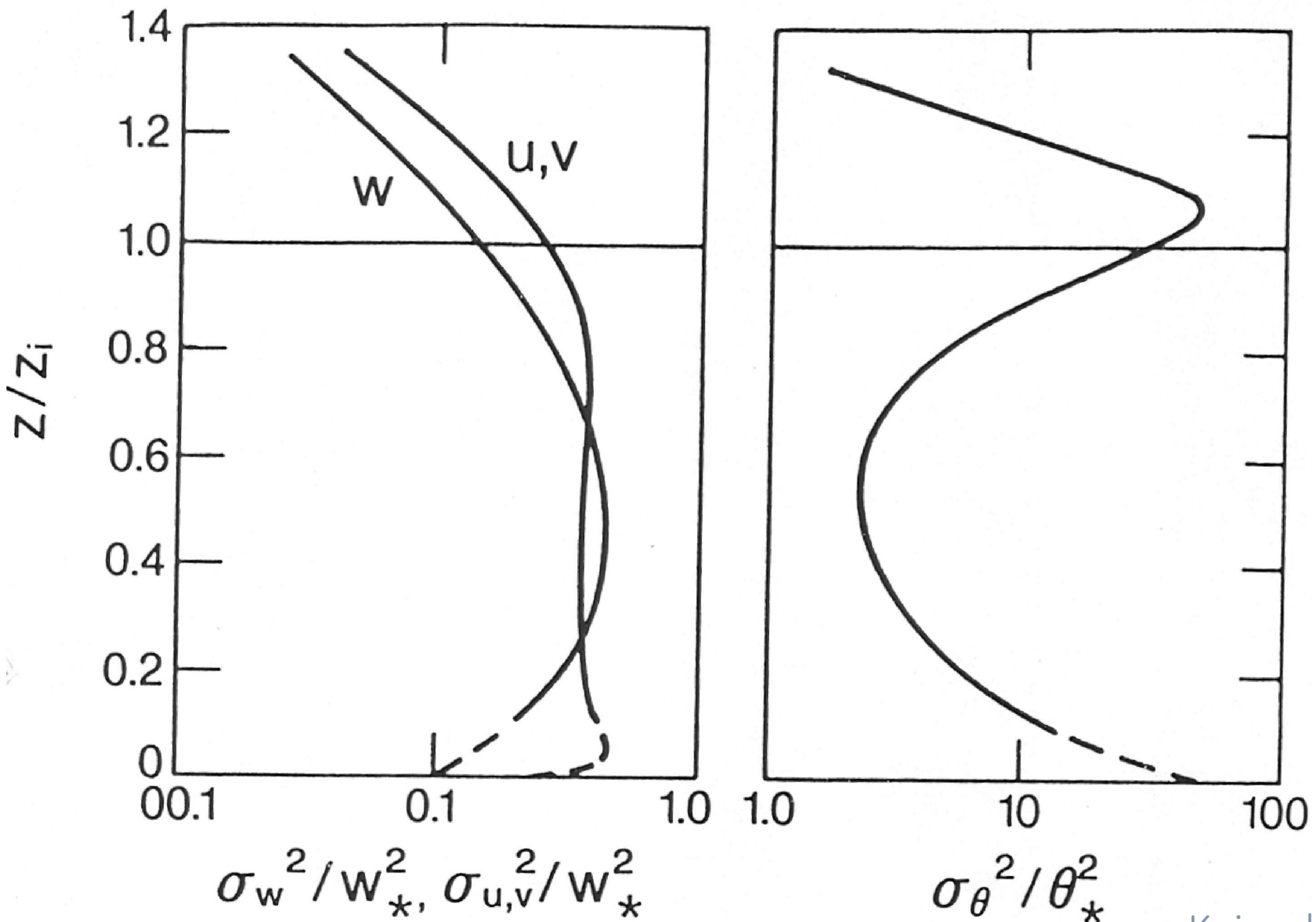
CBL
→ different measurements (symbols)

Scaling regimes: Mixed Layer



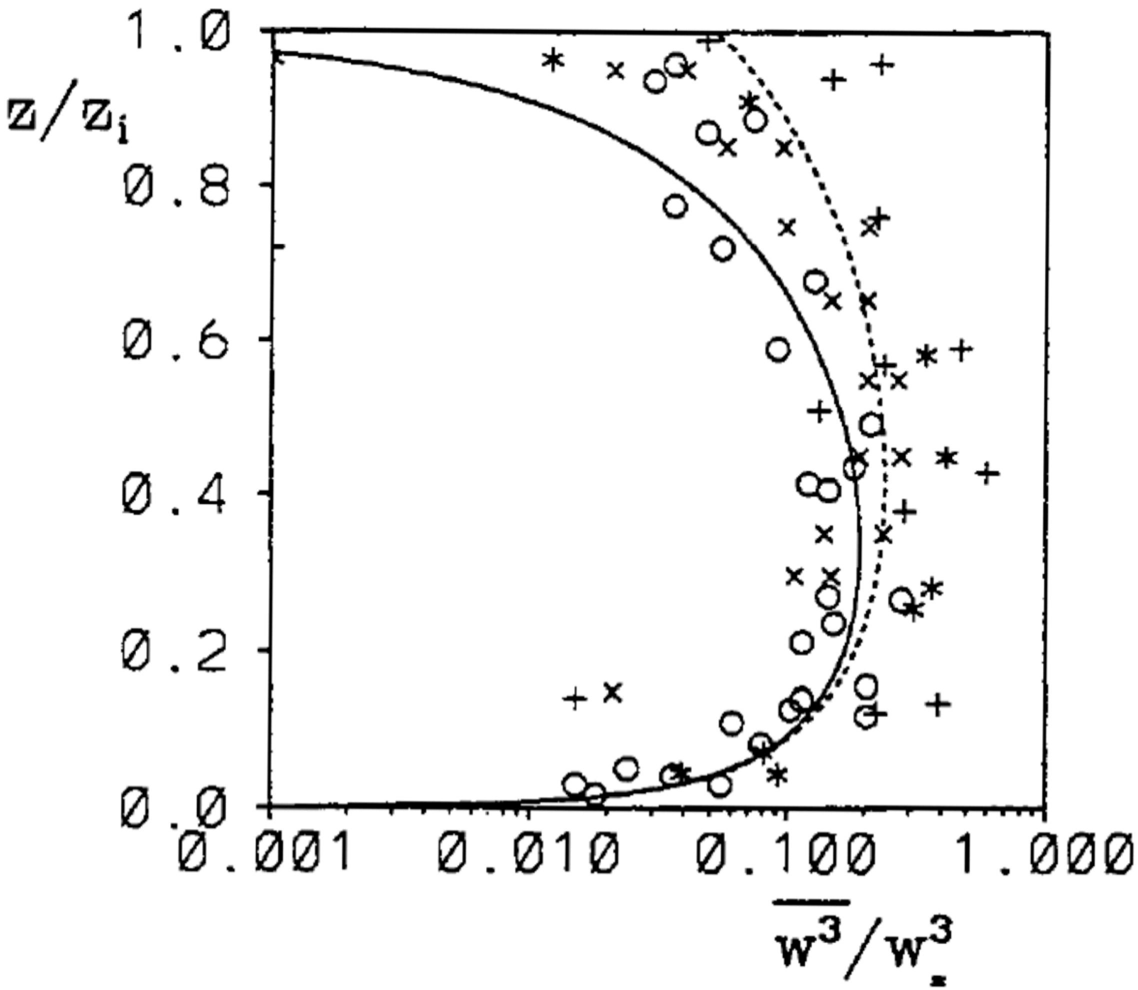
Kaimal and Finnigan (1994)

Scaling regimes: Mixed Layer

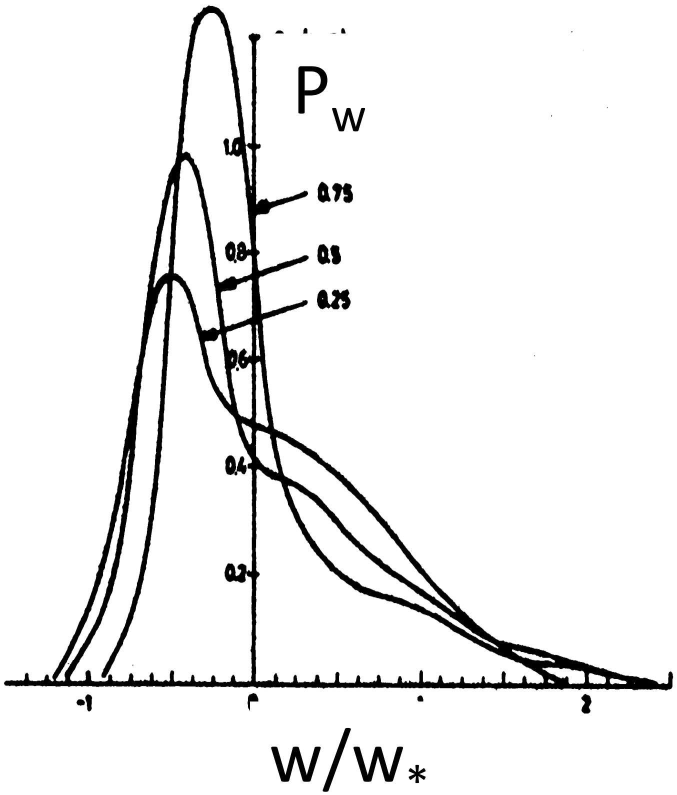


Kaimal and Finnigan (1994)

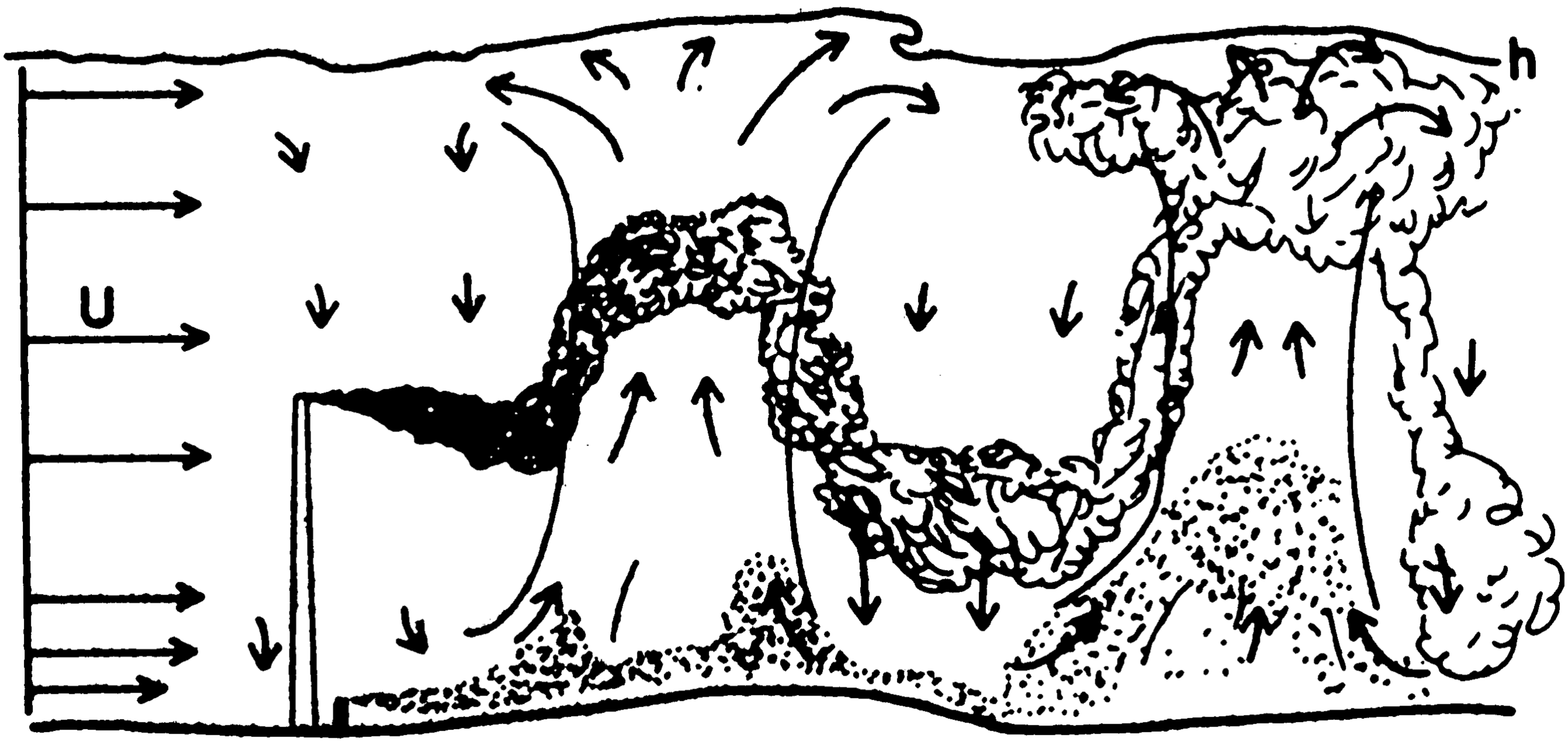
Skewness w: Mixed Layer

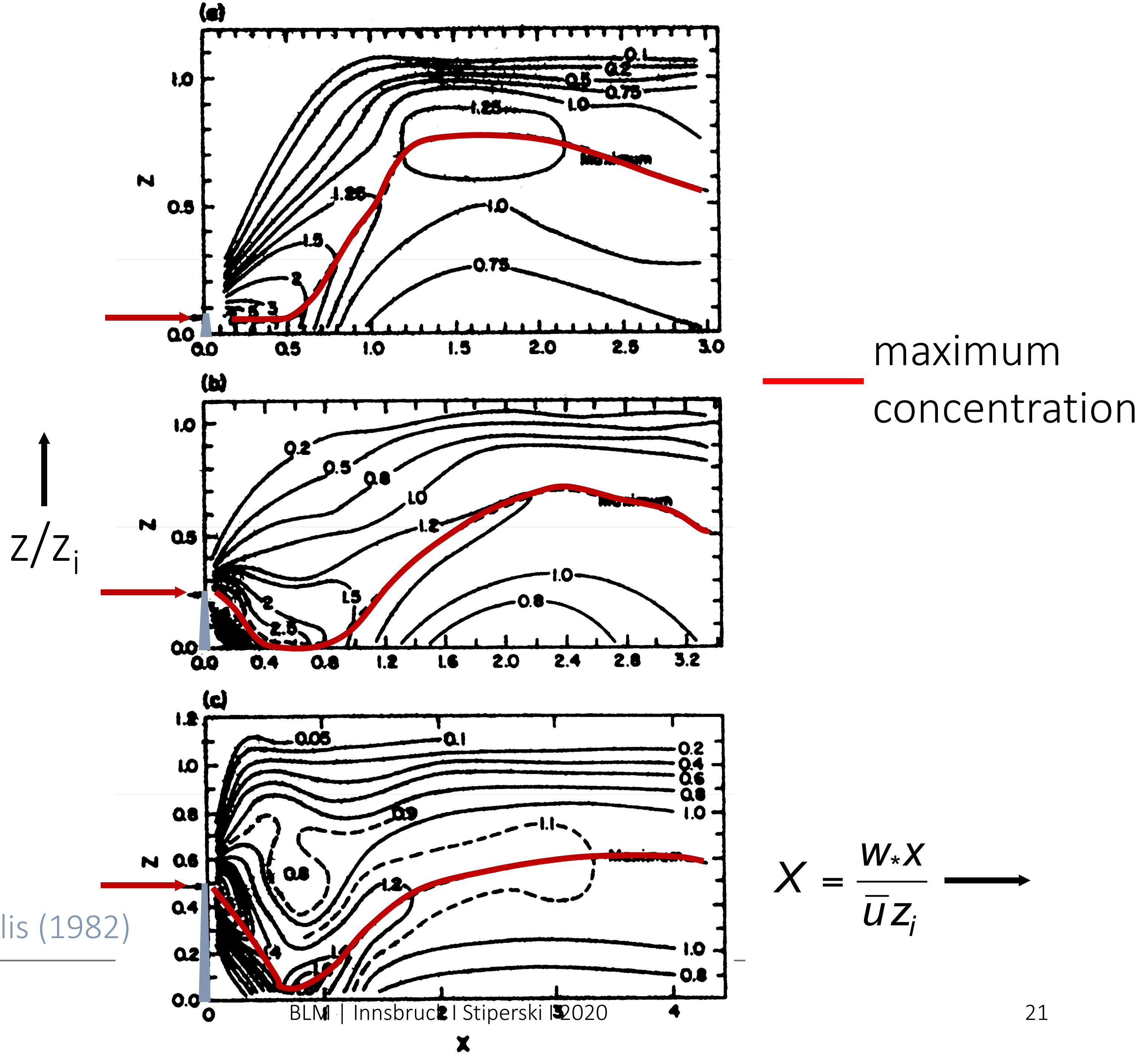


$$\frac{\overline{w^3}}{w_*^3} = f_{w3}\left(\frac{z}{z_i}\right)$$



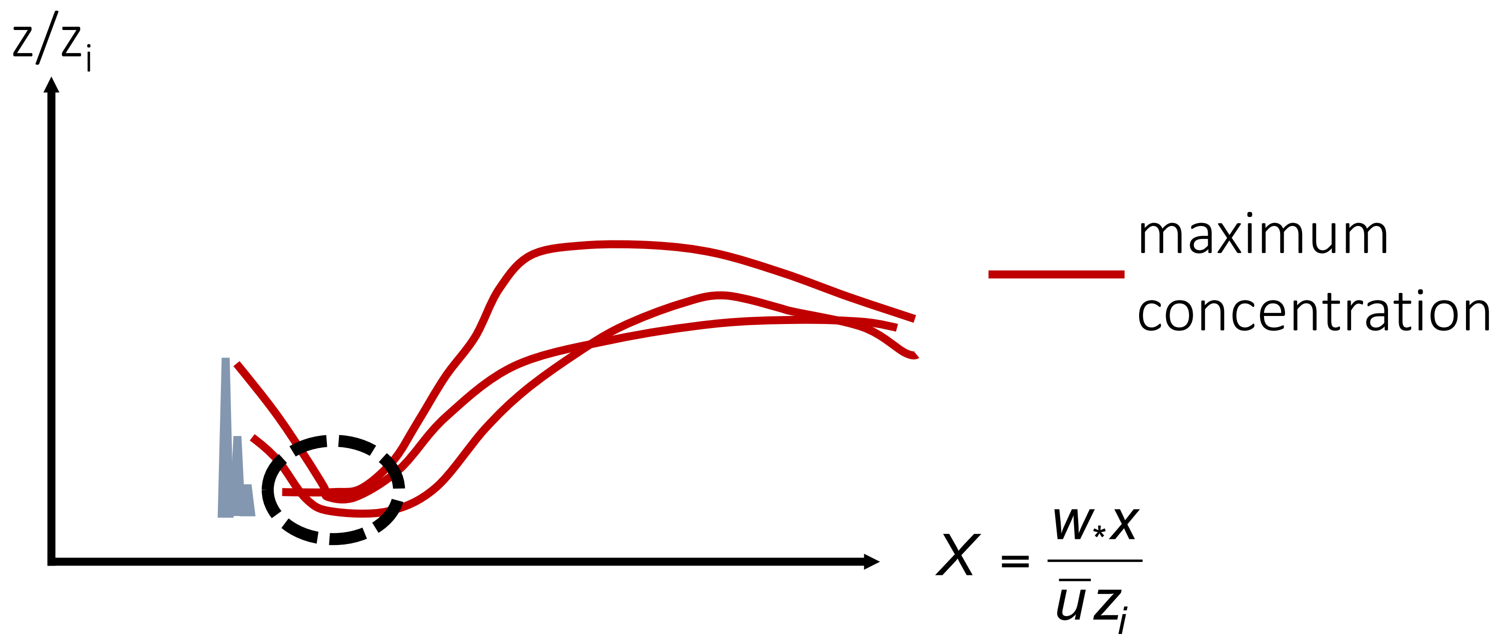
Mixed layer





Deardorff and Willis (1982)

Scaling regimes: Mixed Layer



→ scaling of concentration in ML: not 'similar' for different emission heights

→ not only local dispersion process

→ non-local (large eddies) mixing

Deardorff and Willis (1982)

Summary: Mixed Layer Scaling

→ one π -group: z/z_i

→ every scaled mean variable: $$\frac{\bar{a}}{a_*} = f_a\left(\frac{z}{z_i}\right)$$

→ convective velocity scale: w_*

→ works for wind & scalar variances, turbulent fluxes, skewness w, ..

→ works for spectra (chapter 7)

→ ‘not interesting’ for profiles of mean variables (uniform)

→ does not work for mean concentrations (non-local influence)

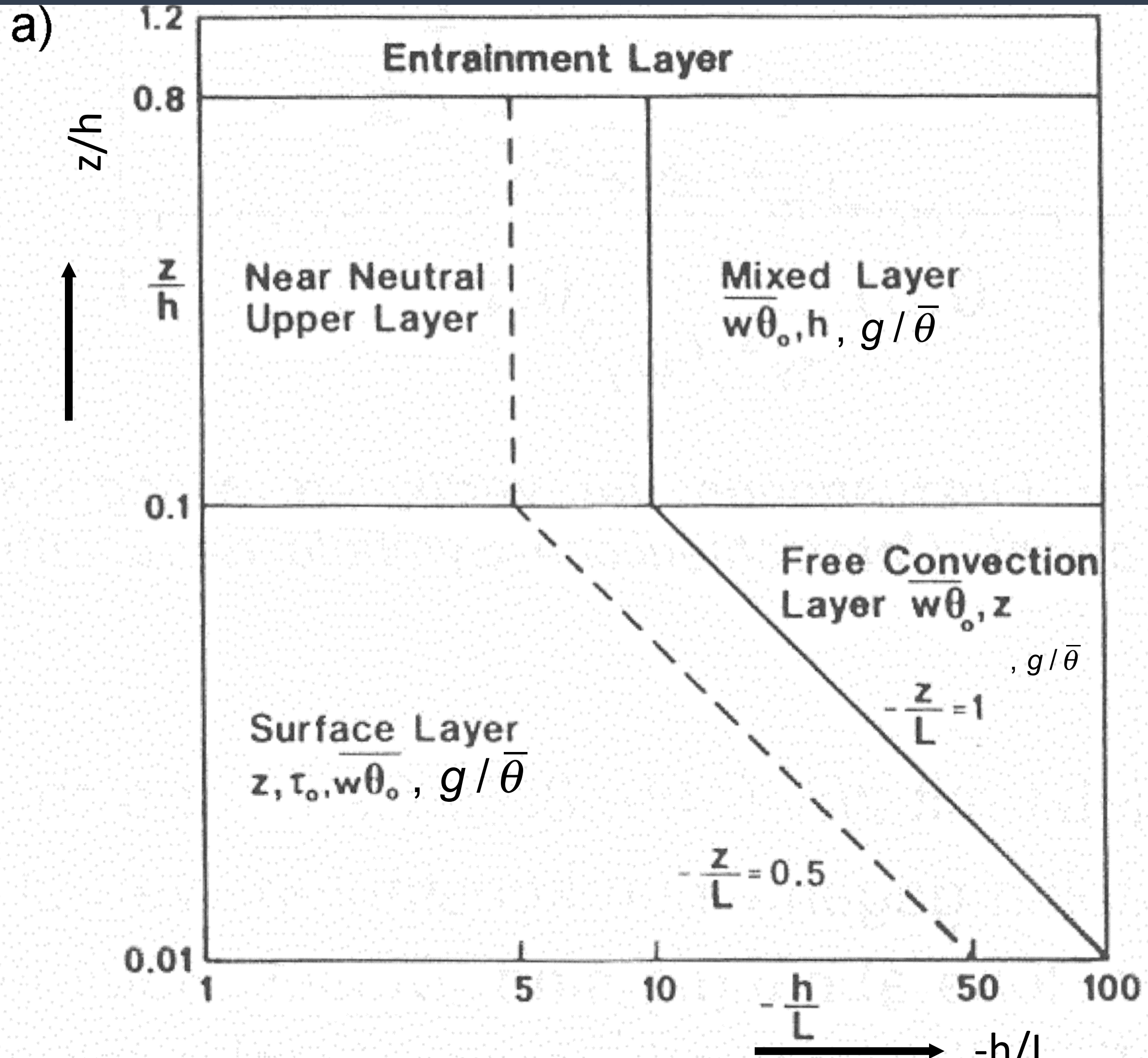
Scaling regimes: The successful Three

→ Surface Layer (MOST) ✓

→ Mixed Layer ✓

→ Local Scaling layer (incl. z-less scaling)

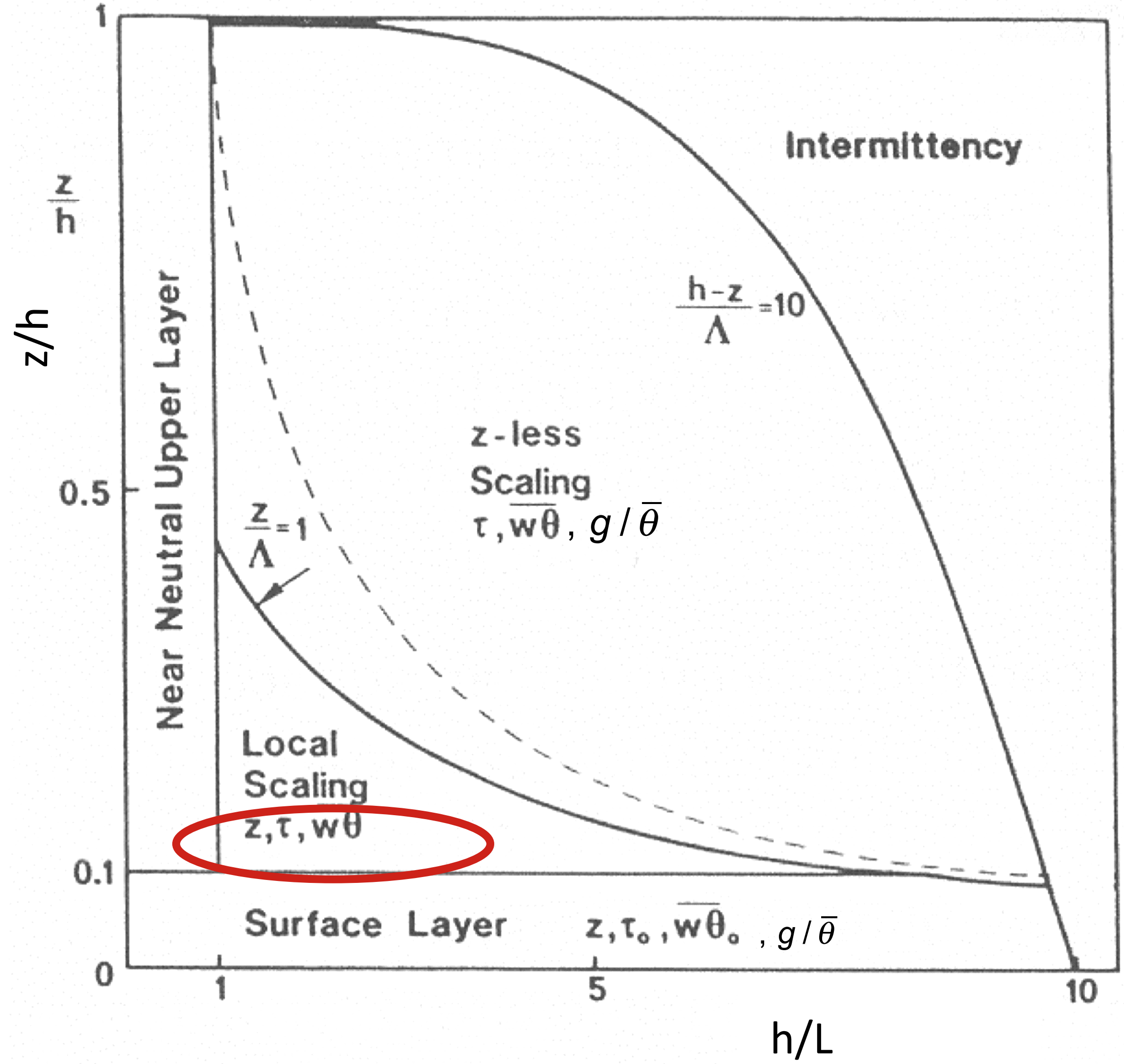
Scaling regimes: Unstable



$h \triangleq z_i$

$$L \equiv -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'_o}}\left(\frac{g}{\overline{\theta}}\right)^{-1}$$

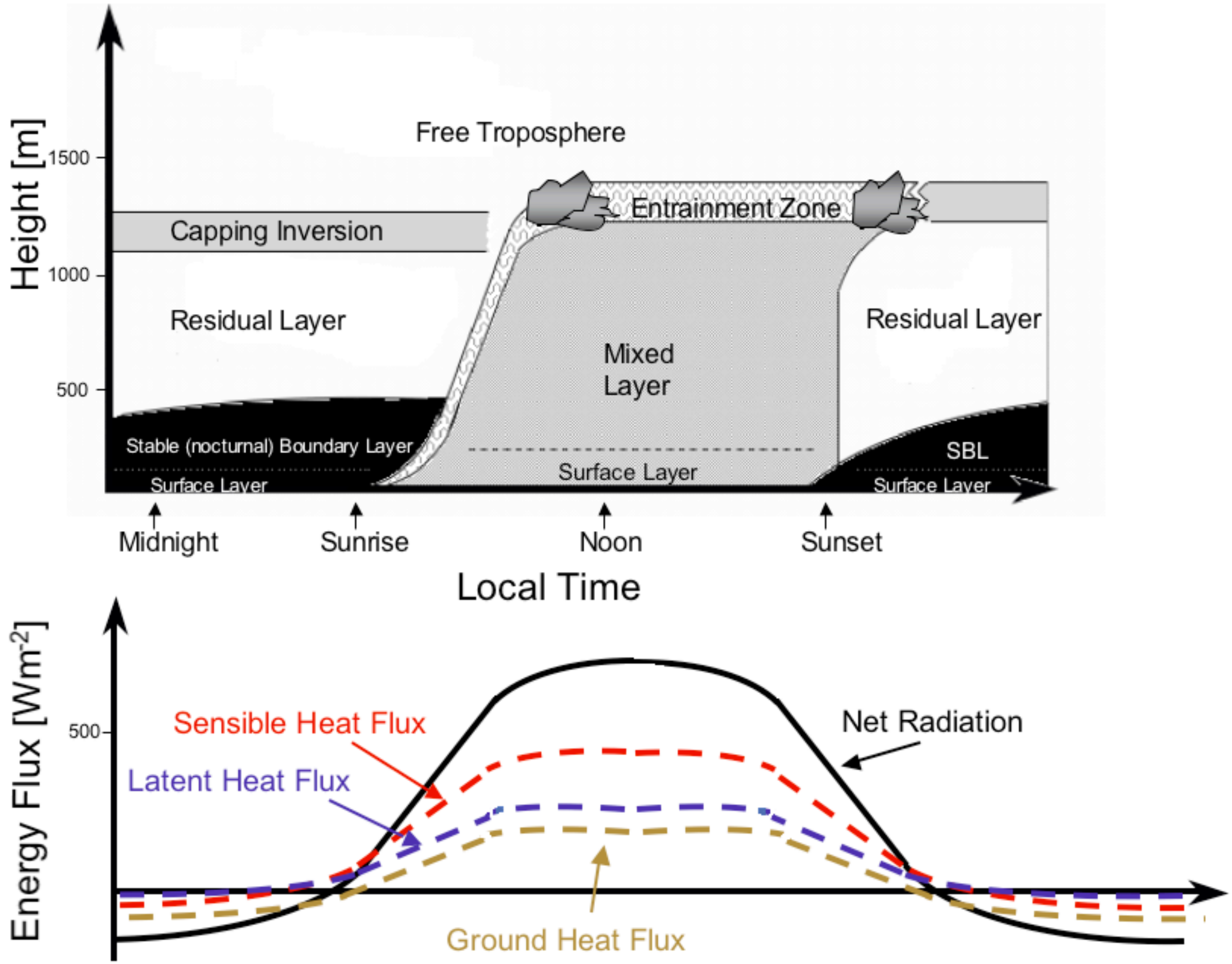
Scaling regimes: Stable



h = SBL height

$$L \equiv -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'_o}}\left(\frac{g}{\overline{\theta}}\right)^{-1}$$

Scaling regimes



Scaling regimes: Local Scaling

→ stable stratification (large z/L)

→ weak turbulence

→ decoupled from surface

→ not the surface fluxes for scaling

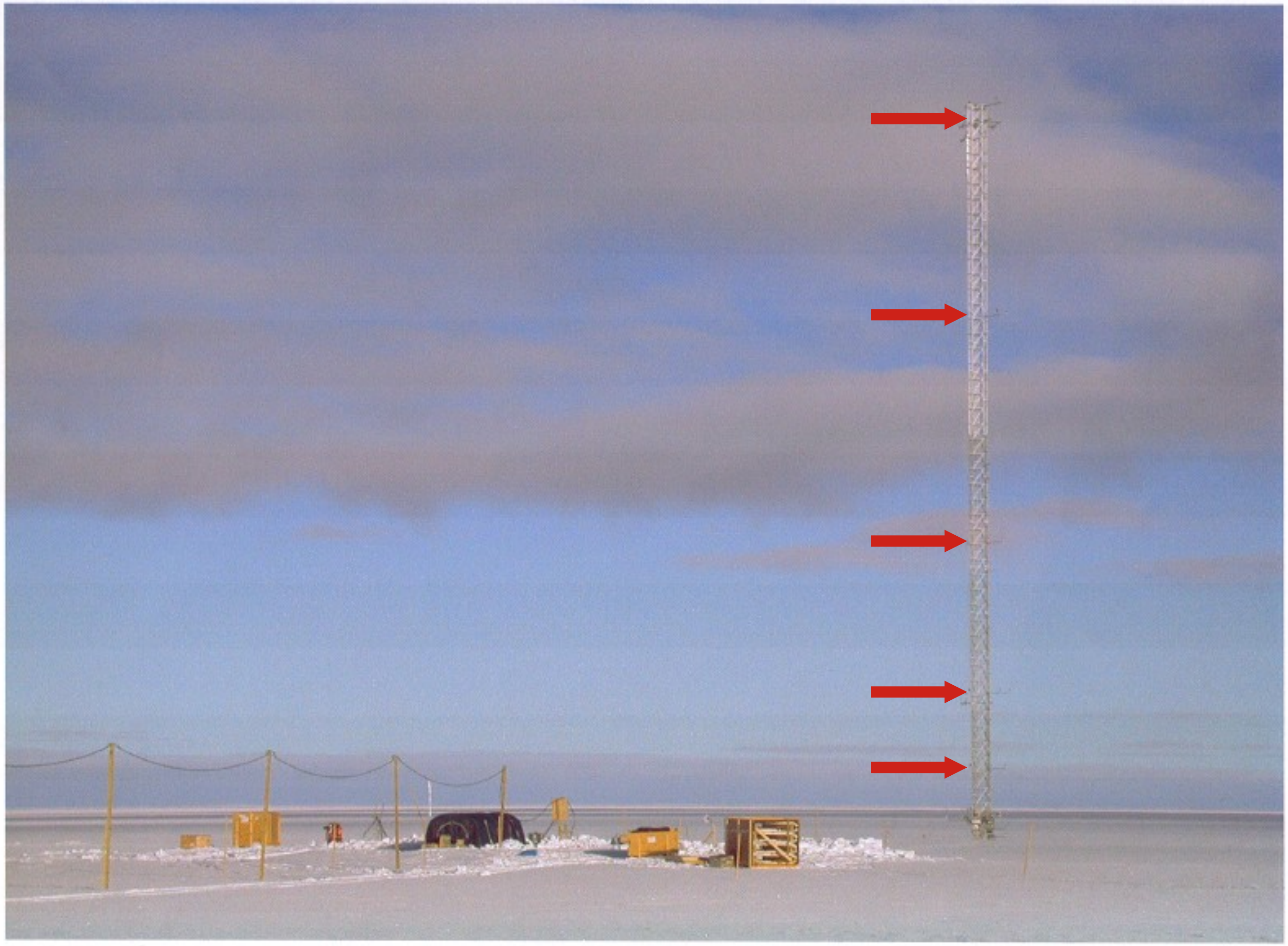
$$\frac{\bar{a}}{a_*} = f_a\left(\frac{z}{\Lambda}\right)$$

MOST		LS
$\overline{u'w'}_o \left[= \tau_o\right]$	→	$\overline{u'w'}(z) \left[= \tau(z)\right]$
$\overline{w'\theta'}_o$	→	$\overline{w'\theta'}(z)$
L	→	$\Lambda = \dfrac{\tau^{3/2}}{k \frac{g}{\bar{\theta}} \overline{w'\theta'}}$

local scaling variables

Scaling regimes: Local Scaling

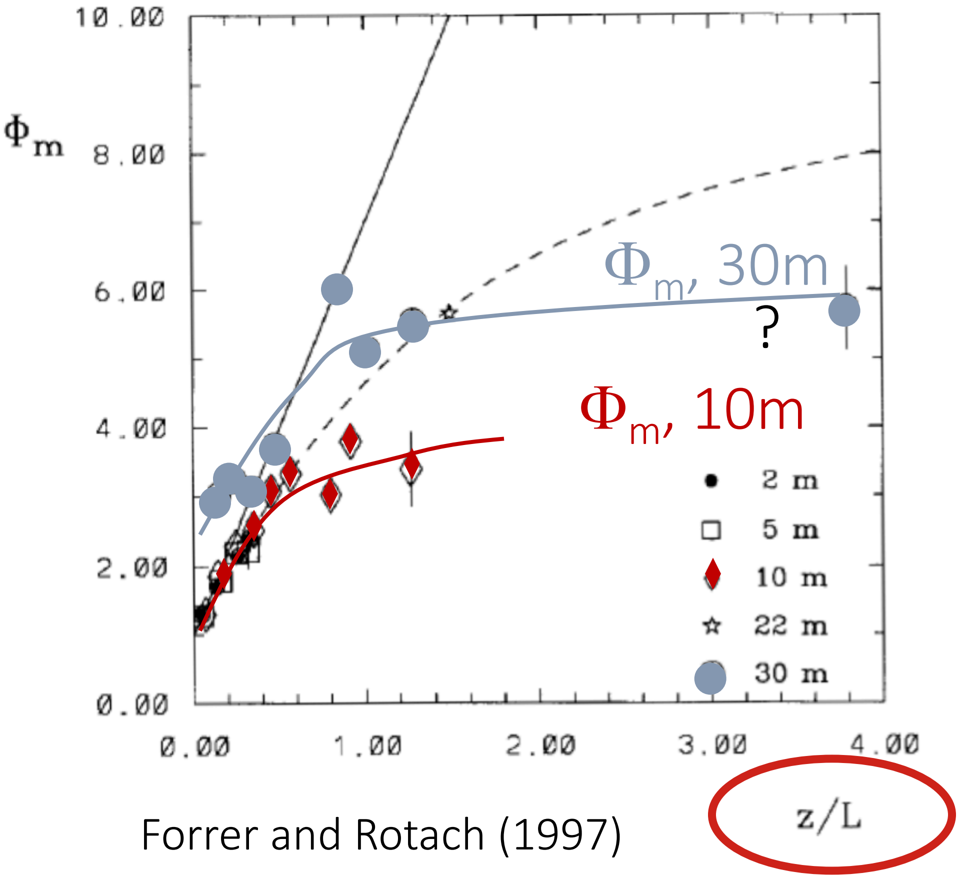
→ Greenland, non-dimensional wind shear



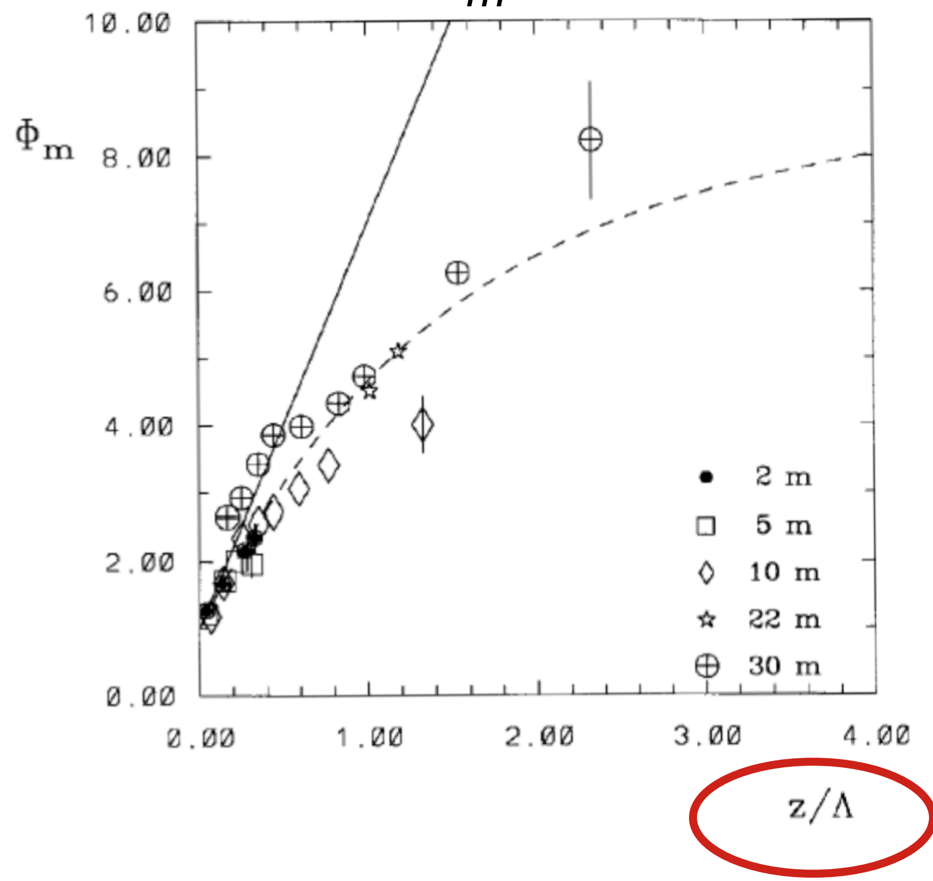
$$\frac{kz}{u_*}\frac{\partial \bar{u}}{\partial z} = \phi_m$$

Scaling regimes: Local Scaling

→ Greenland, non-dimensional wind shear ϕ_m



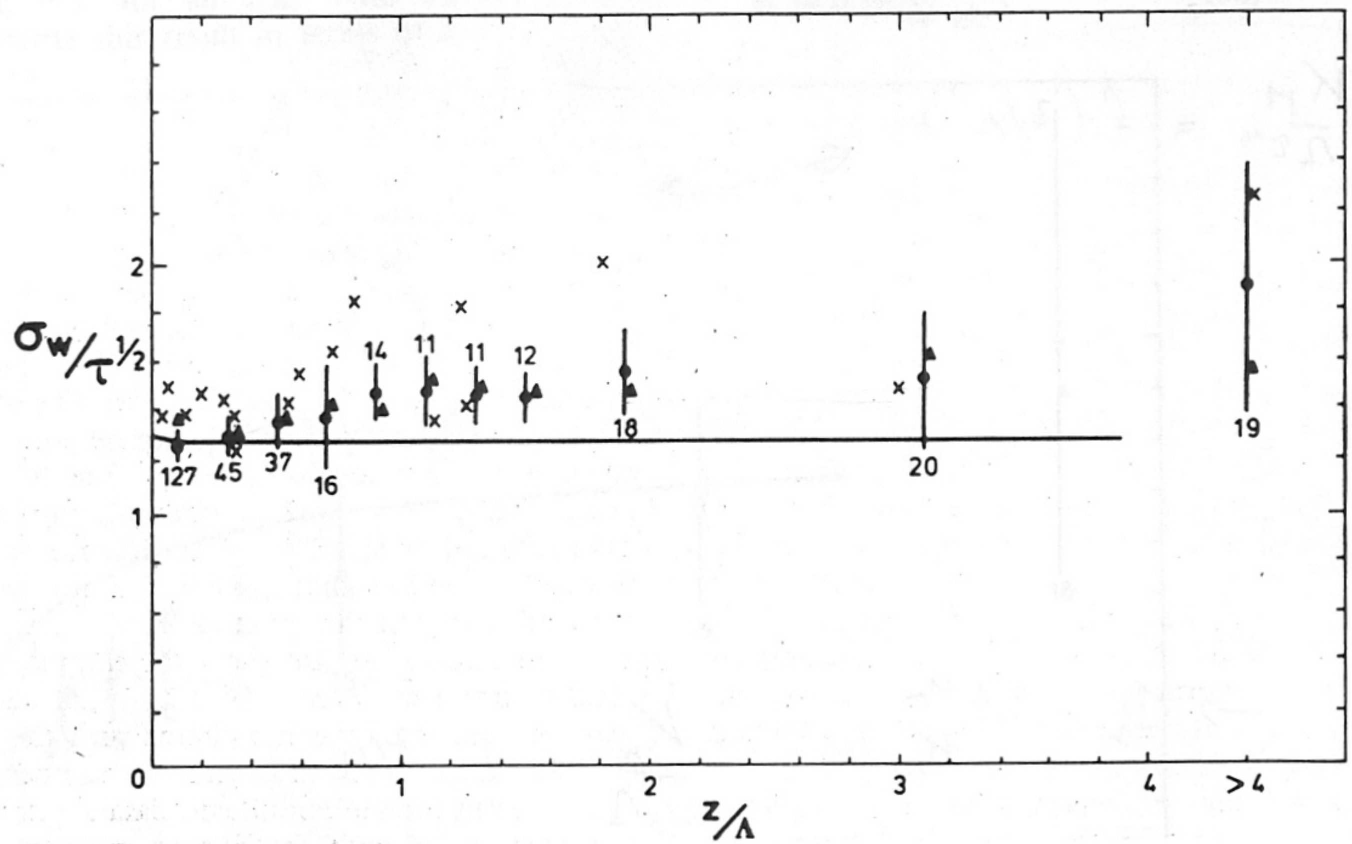
Forrer and Rotach (1997)



- - - - - - - Beljaars and Holtslag, 1991

——— Businger et al. 1971

Scaling regimes: Local Scaling

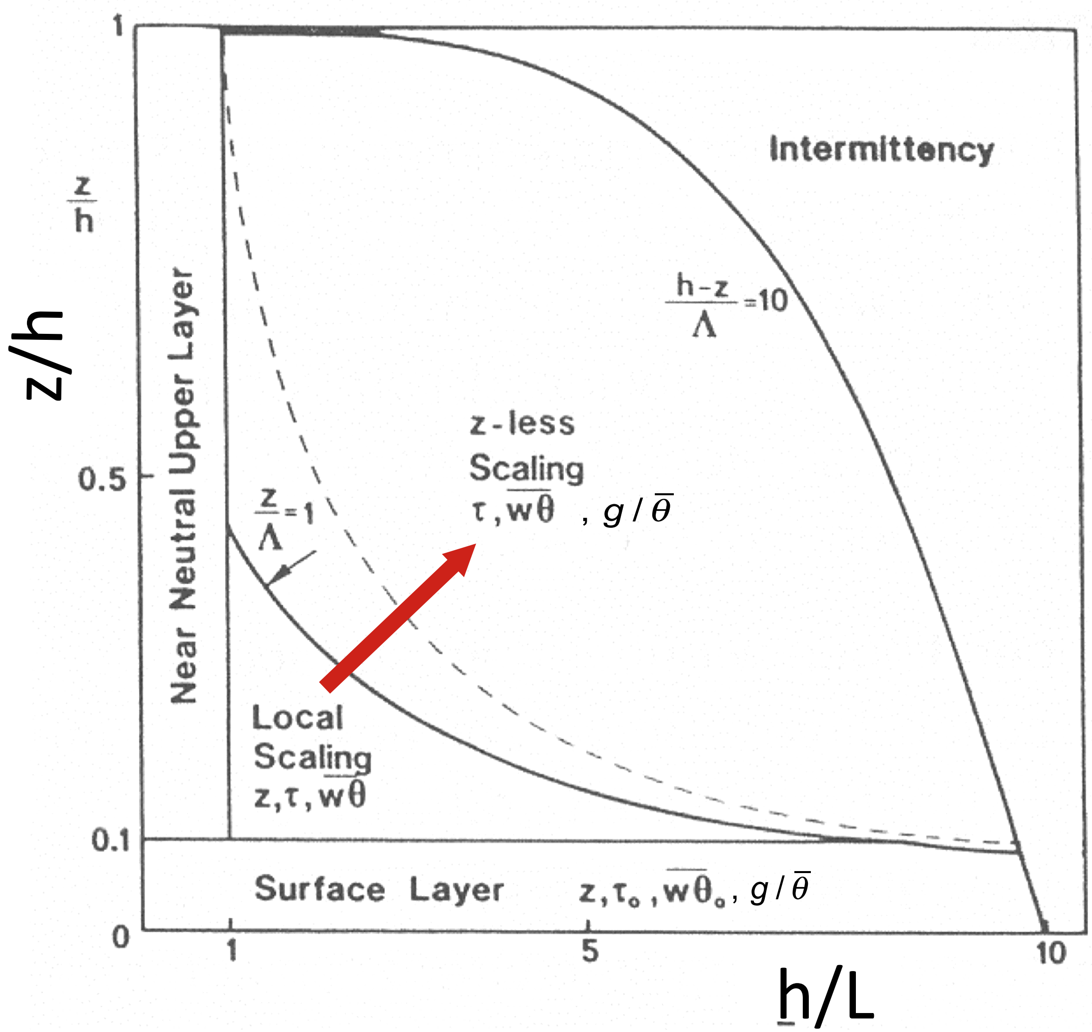


Nieuwstadt (1984)

Local Scaling: z-less limit

h = SBL height

$$L \equiv -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'_o}}\left(\frac{g}{\overline{\theta}}\right)^{-1}$$



Scaling regimes: z-less Scaling

z-less limit: height z no longer important

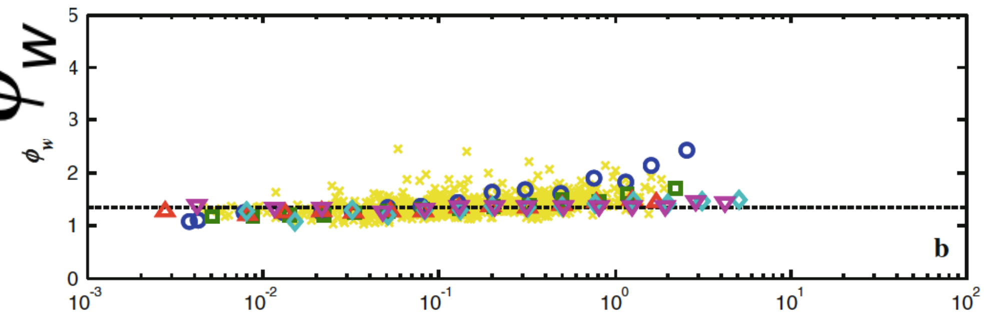
→ stability so strong that surface influence vanishes

→ Consequences:

Flux-variance relations = const

$$\frac{\sigma_w}{u_{*\ell}}(=\phi_w(z/\Lambda)=const$$

ϕ_w



Grachev et al. (2013)

$\rightarrow z/\Lambda$

Scaling regimes: z-less Scaling

z-less limit: height z no longer important

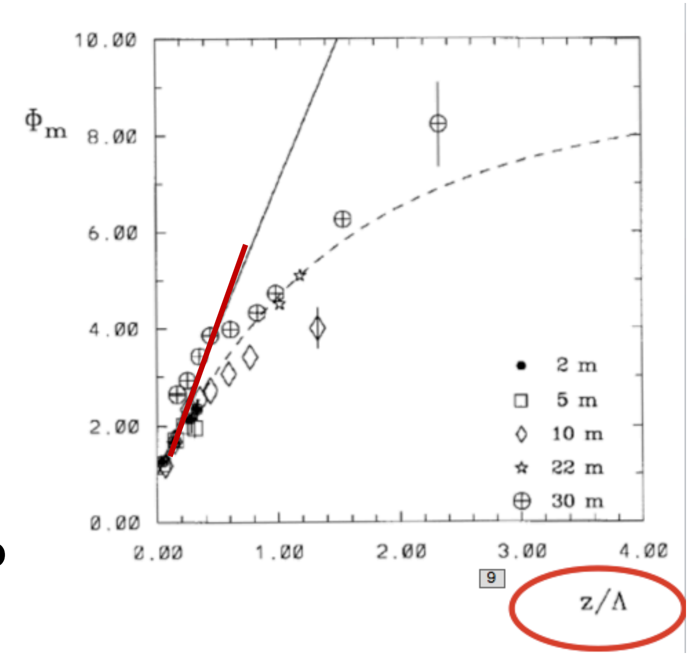
→ stability so strong that surface influence vanishes

→ Consequences:

Flux-gradient relations = linear

$$\frac{\partial \bar{u}}{\partial z}\frac{kz}{u_{*_\ell}}(=\phi_m(z/\Lambda)) = const.\cdot\frac{z}{\Lambda}$$

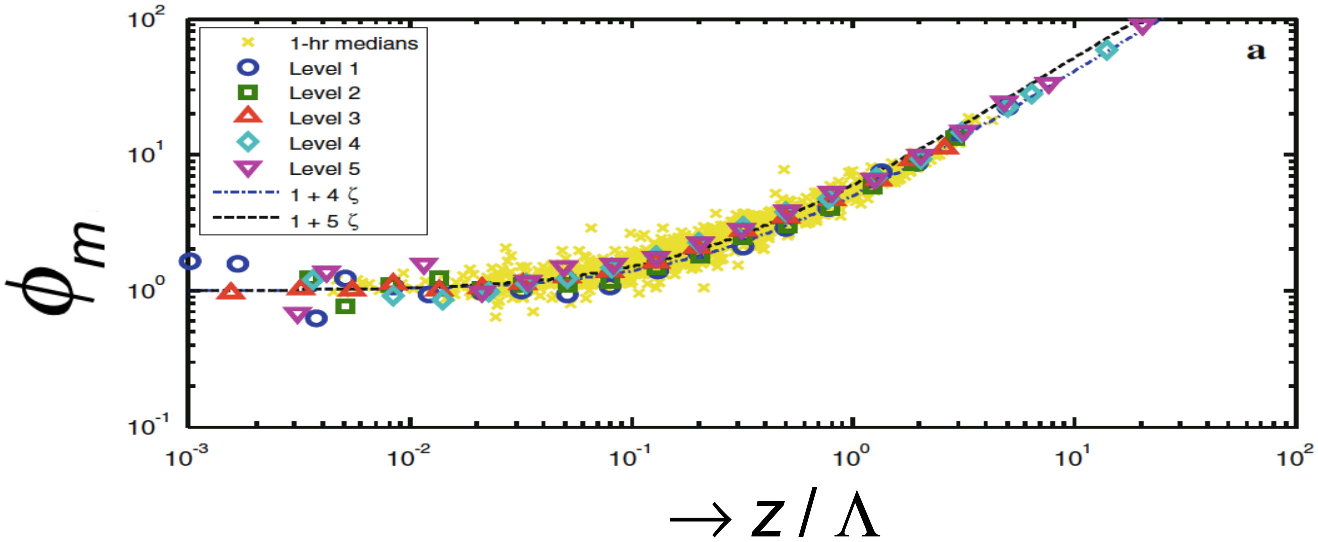
?



Scaling regimes: z-less Scaling

z-less limit: height z no longer important

$$\frac{\partial \bar{u}}{\partial z}\frac{kz}{u_{*_\ell}} = 1 + \beta\frac{z}{\Lambda}$$



→ **z-less**: only if $R_f < 0.25$ and $Ri < 0.25$

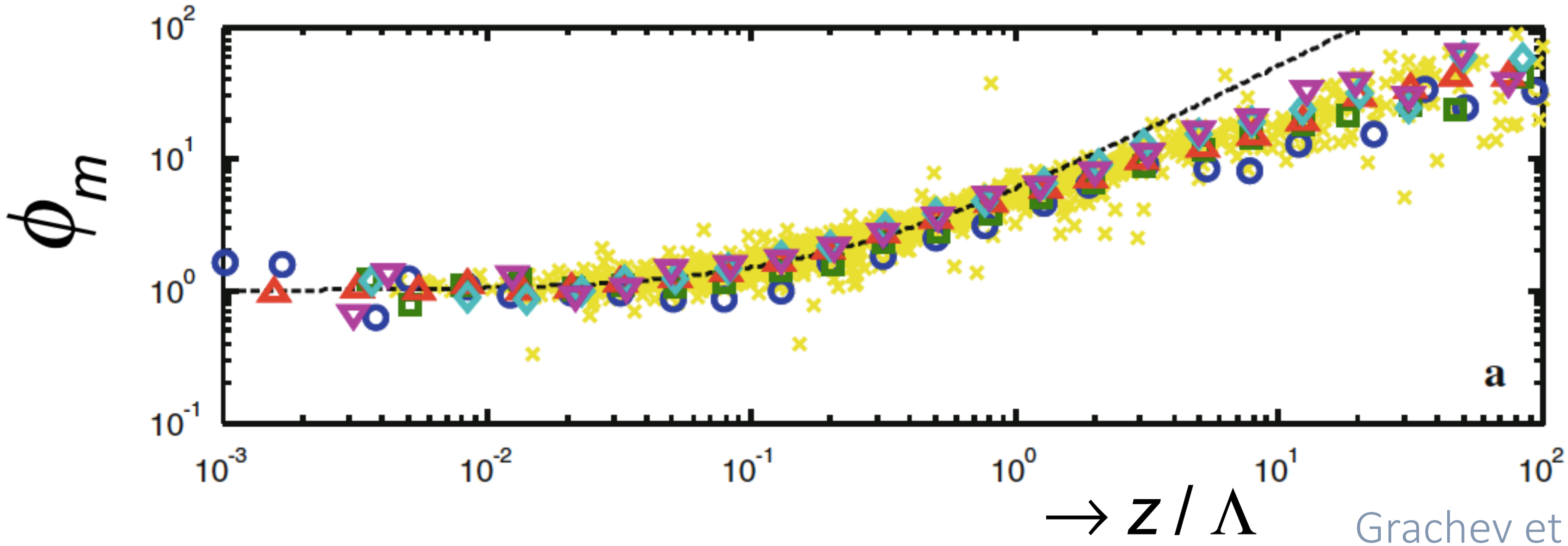
→ 'well-behaved' (fully developed) turbulence

Grachev et al. (2013)

Scaling regimes: z-less Scaling

z-less limit: height z no longer important

$$\frac{\partial \bar{u}}{\partial z}\frac{kz}{u_{*_\ell}} = 1 + \beta \frac{z}{\Lambda} \quad \longleftrightarrow \quad \text{fully developed turbulence}$$

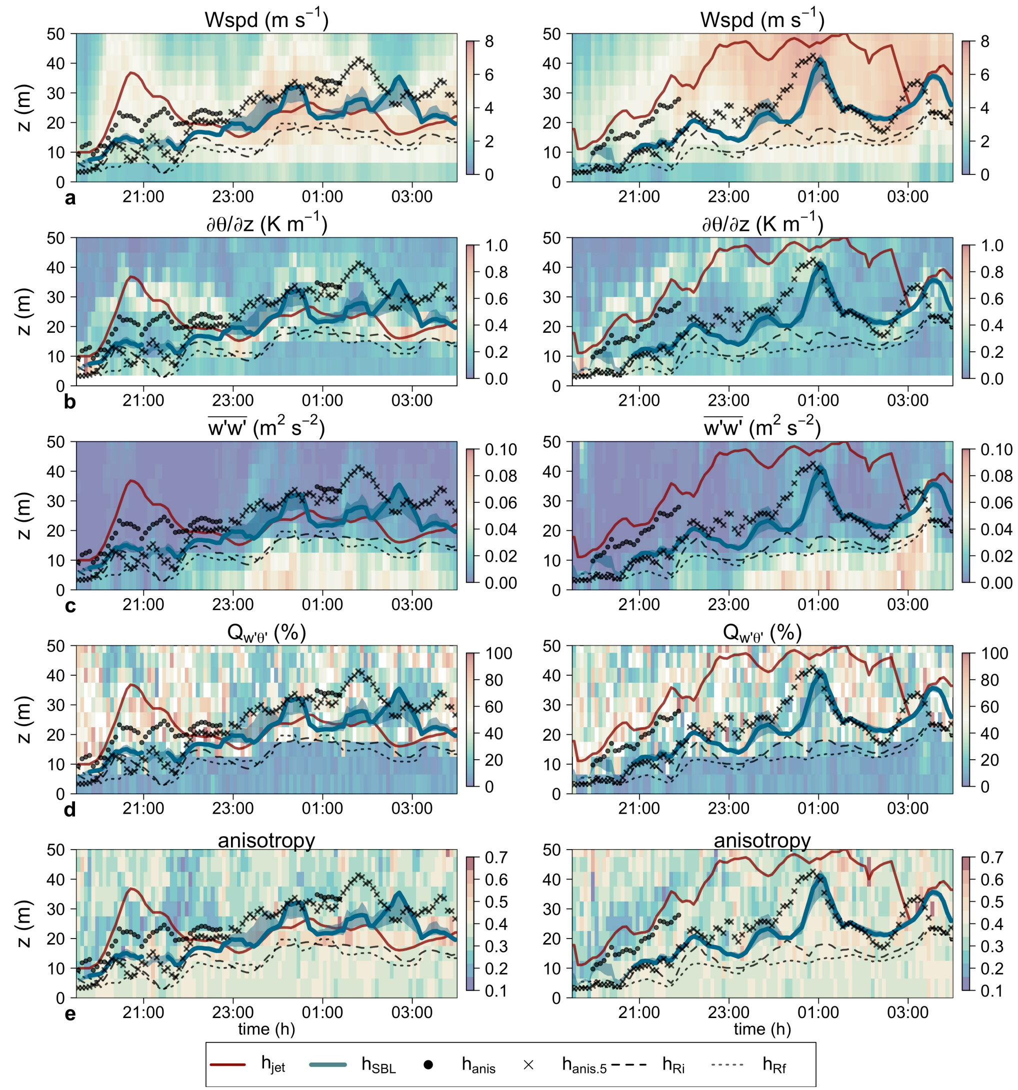


Grachev et al. (2013)

→ for 'all conditions' (Ri, R_f): closer to Beljaars and Holtslag formulation

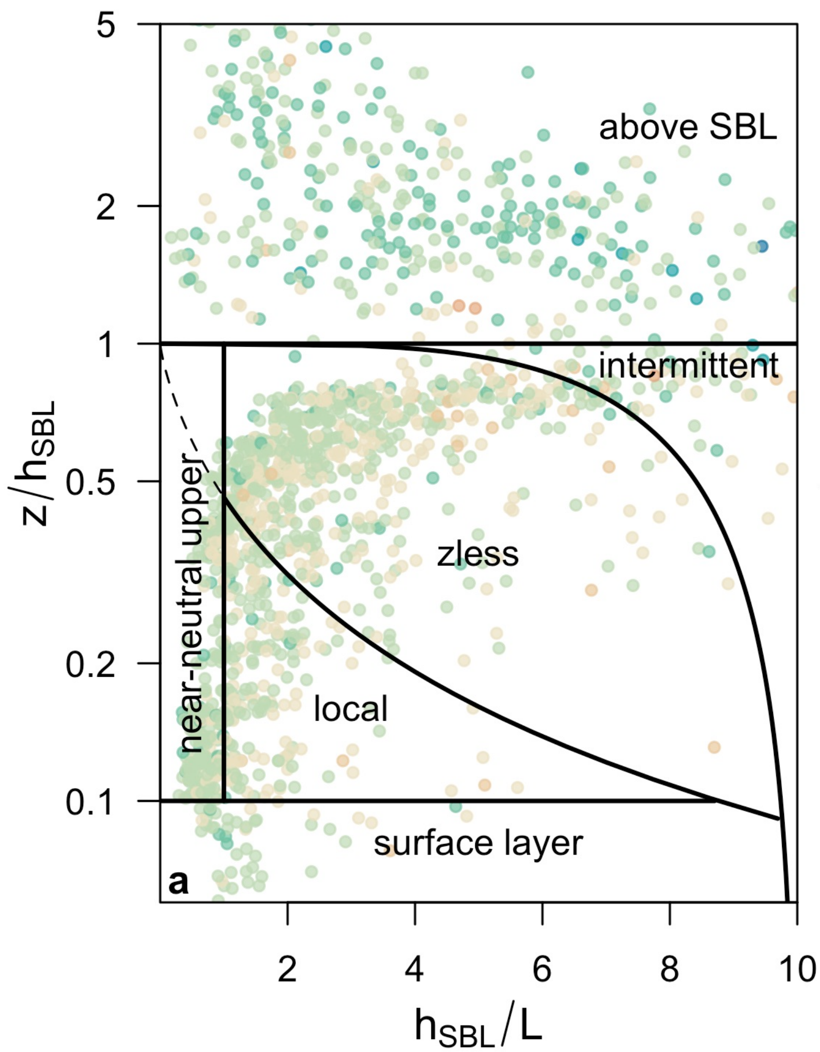
→ deviation from 'z-less' behavior

Stable scaling regimes: shallow slope



- 50 m tower on a gentle slope (1°)
- Katabatic flows form
- Top of PBL can be detected from tower measurements
- Ambiguity of PBL top estimate
- But we can test scaling regimes

Stable scaling regimes: shallow slope

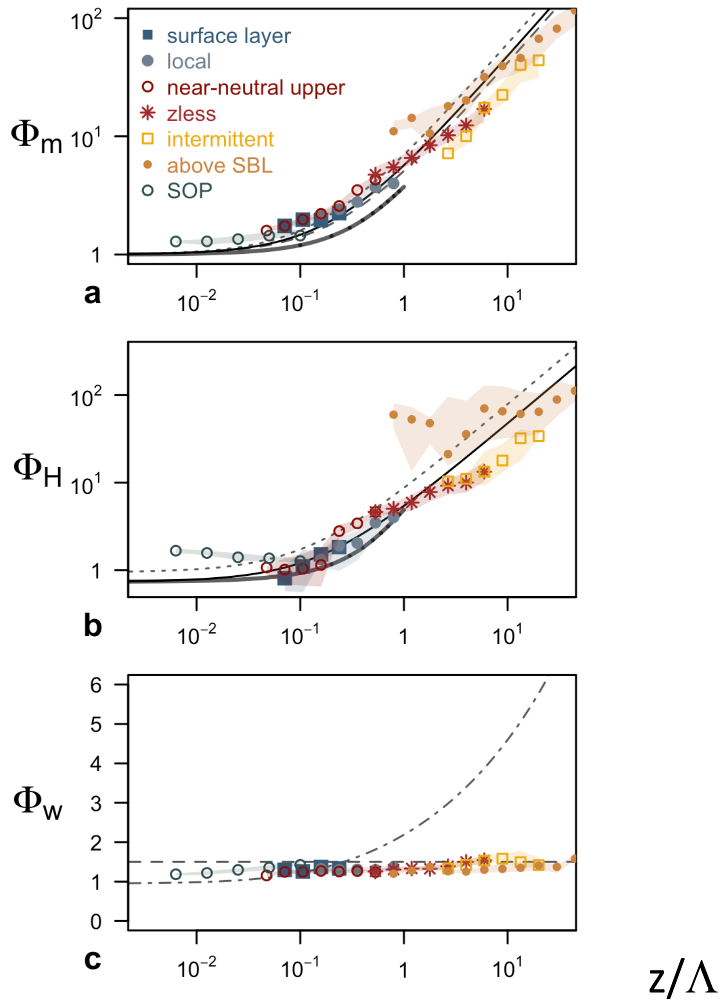


Data from all scaling regimes:

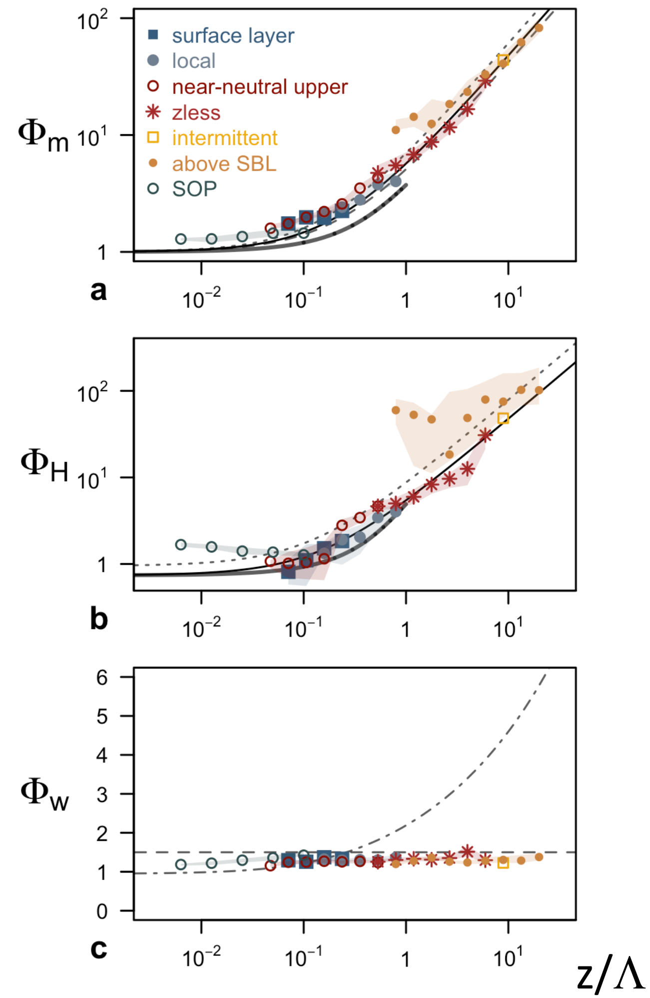
- Surface layer (1st level) – for high h
- Local scaling
- z-less scaling
- Intermittent
- Above SBL

Stable scaling regimes: shallow slope

- Stationary data from each regime



- Stationary & Ri < 0.21

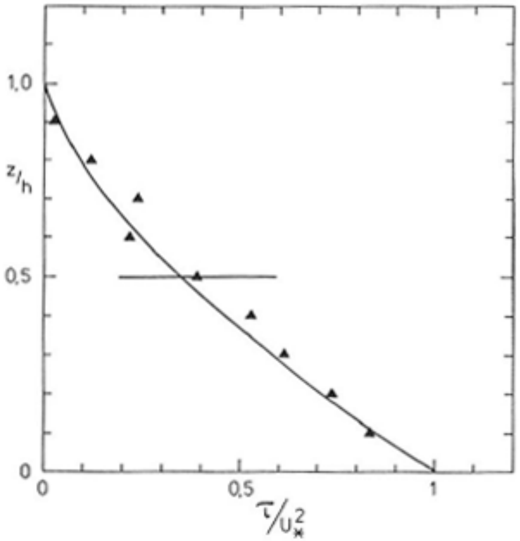


Scaling regimes: Local Scaling

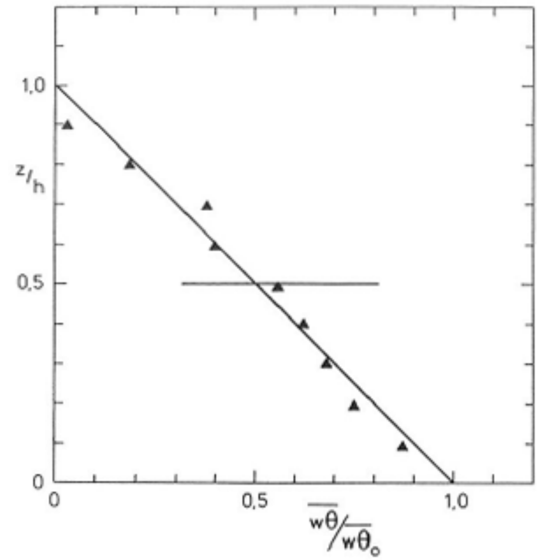
For **Local scaling** to be successful:

Surface fluxes → we need local flux profiles (h = SBL height)

[theoretical derivation (local scaling), Nieuwstadt (1984)]



$$\frac{\tau}{\tau_o} = \left(1 - \frac{z}{h}\right)^{3/2}$$

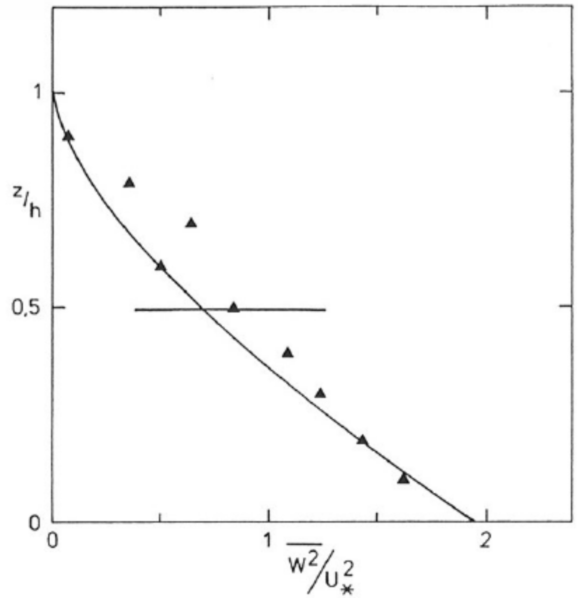


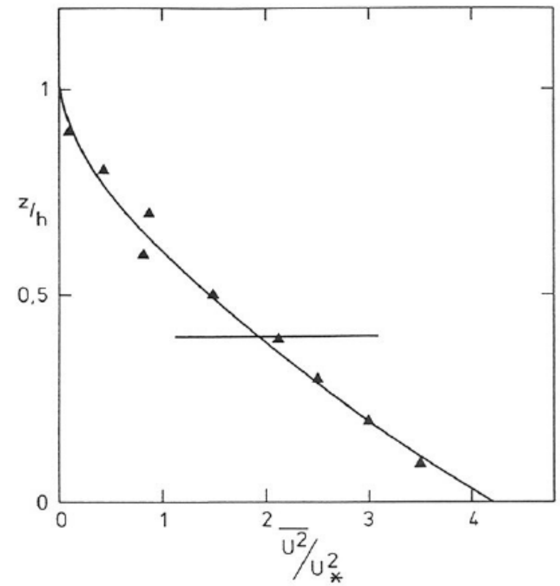
$$\frac{\overline{w'\theta'}}{\overline{w'\theta'}_o} = \left(1 - \frac{z}{h}\right)$$

Nieuwstadt (1984)

Scaling regimes: Local Scaling

→ profiles of variances





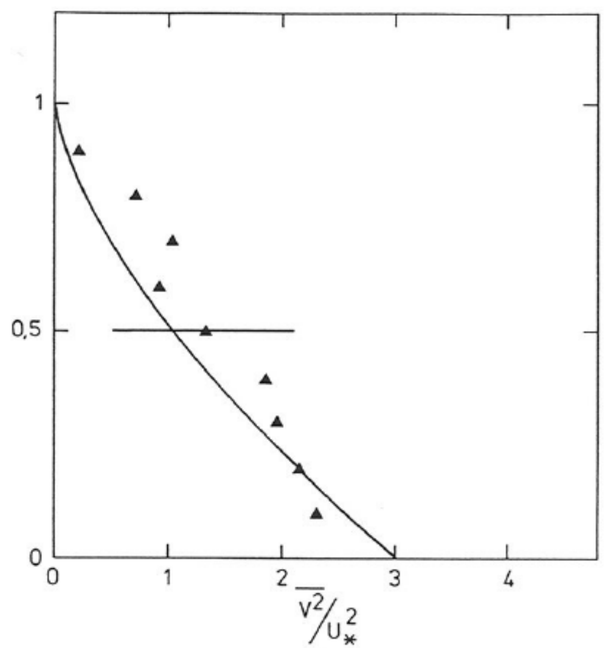
$$\frac{\sigma_w}{u_*} = 1.4\left(1 - \frac{z}{h}\right)^{3/4}$$

$$\frac{\sigma_u}{u_*} = 2.04\left(1 - \frac{z}{h}\right)^{3/4}$$

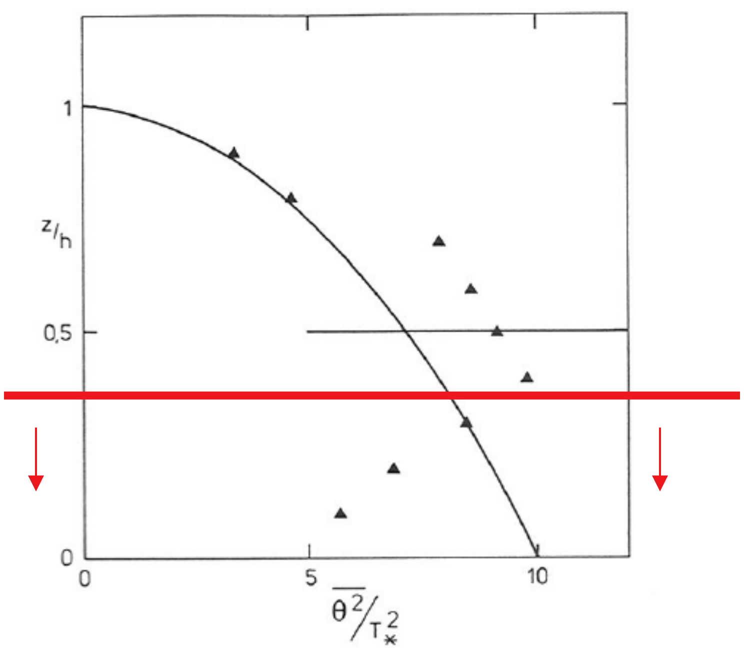
Nieuwstadt (1984)

Scaling regimes: Local Scaling

→ profiles of variances



$$\frac{\sigma_v}{u_*} = 1.7\left(1 - \frac{z}{h}\right)^{3/4}$$



$$\frac{\sigma_\theta}{\theta_*} = 3.2\left(1 - \frac{z}{h}\right)^{1/4}$$

Nieuwstadt (1984)

Summary: Local Scaling

→ one π-group: z / Λ

→ every scaled mean variable: $$\frac{\bar{a}}{a_*} = f_a\left(\frac{z}{\Lambda}\right)$$

→ works for wind & turbulent fluxes

→ works for spectra (Chapter 7)

→ apparently difficult to assess (weak turbulence)

→ does not seem to work for temperature variance

→ subtle border between strongly stable (→ intermittency) and local / z-less scaling

Scaling regimes

- successful $\leftrightarrow$ quasi-stationary
- Surface Layer, Mixed Layer, Local Scaling
- 'mean' = characteristic profiles
- dependent on a few characteristic variables

→ turbulent fluxes: $\overline{u'w'}_{(o)}$ $\overline{w'\theta'}_{(o)}$

→ length scales: z_i, L, Λ

Scaling in complex terrain

→ now that scaling regimes are established in homogeneous terrain...

→ how about completely different conditions?

- not horizontally homogeneous
- not flat

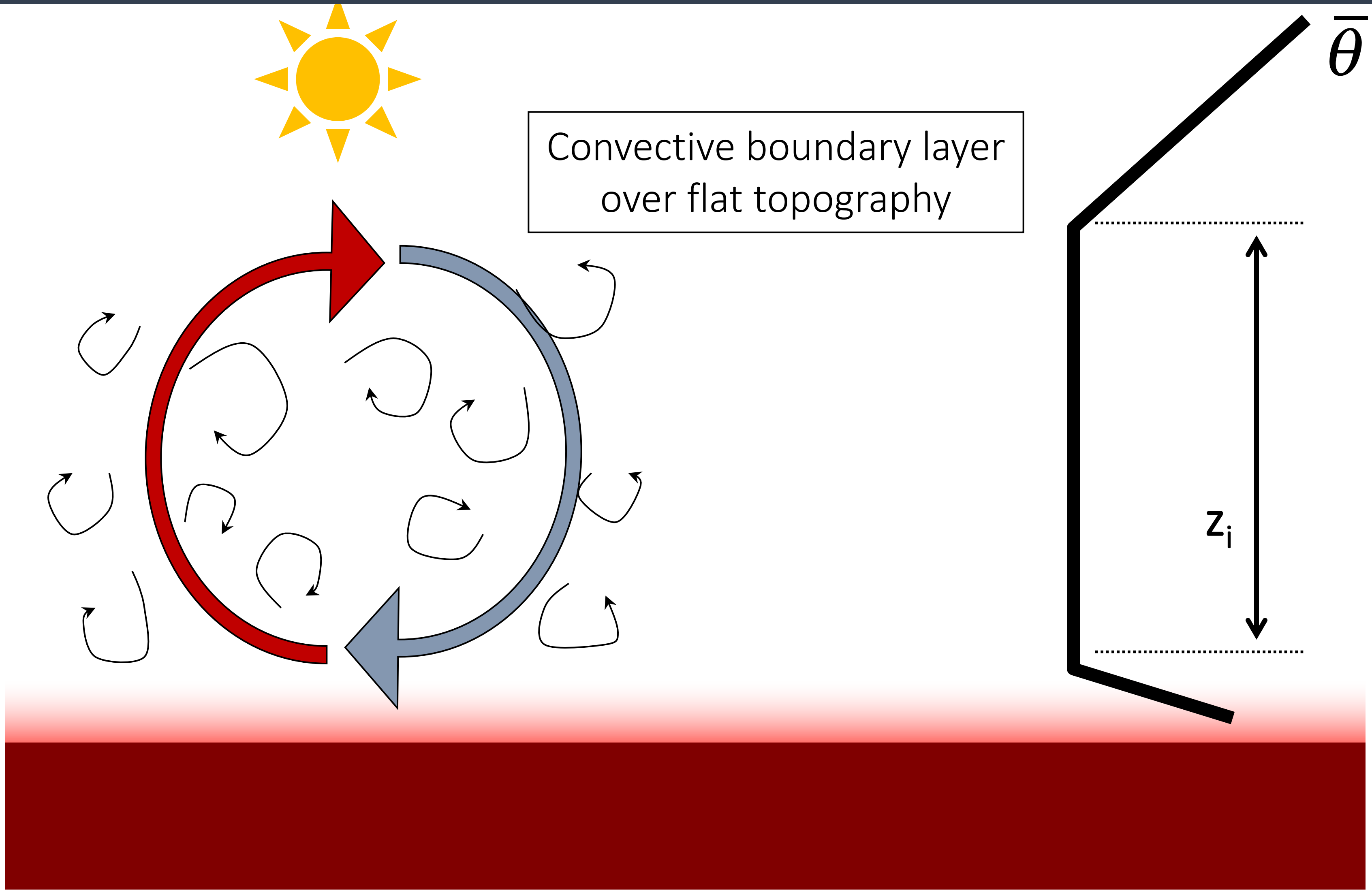
→ do we consider all relevant processes?

Scaling in complex terrain

Application Examples:

1. Mixed-layer TKE scaling (MAP Riviera)
2. Wind directional change with height
3. Local scaling flux-variance relationships (12 Datasets)

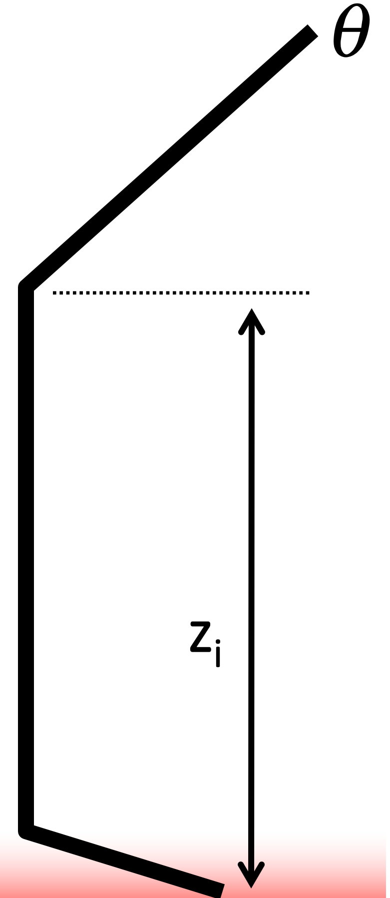
1. Mixed layer scaling in complex terrain



Mixed Layer Scaling

Deardorff (1970,1972)

- example: $$\frac{TKE}{w_*^{\,2}} = f_{TKE}\left(\frac{z}{z_i}\right)$$
- velocity scale $$w_* = \left(\overline{w'\theta'}_o \frac{g}{\overline{\theta}} z_i\right)^{1/3}$$

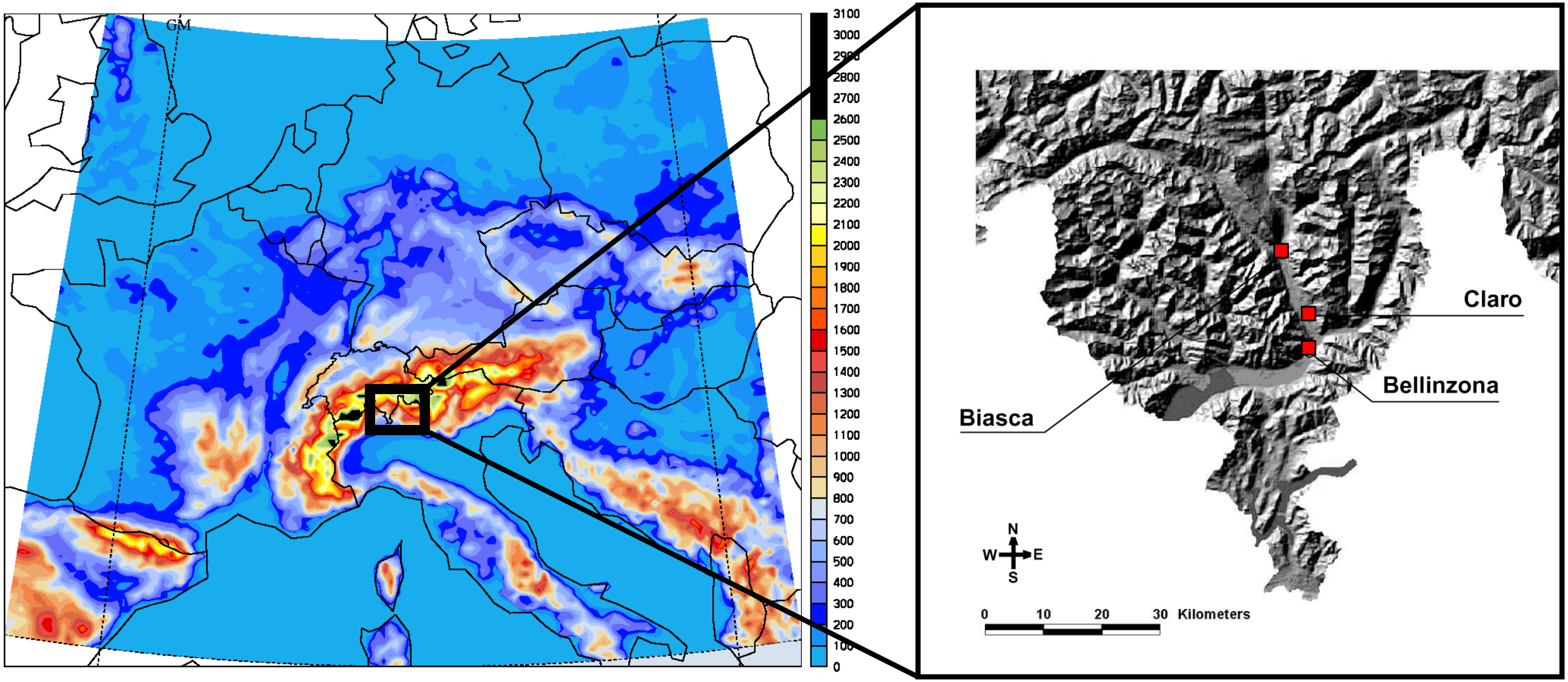


Sensible surface heat flux: H_o

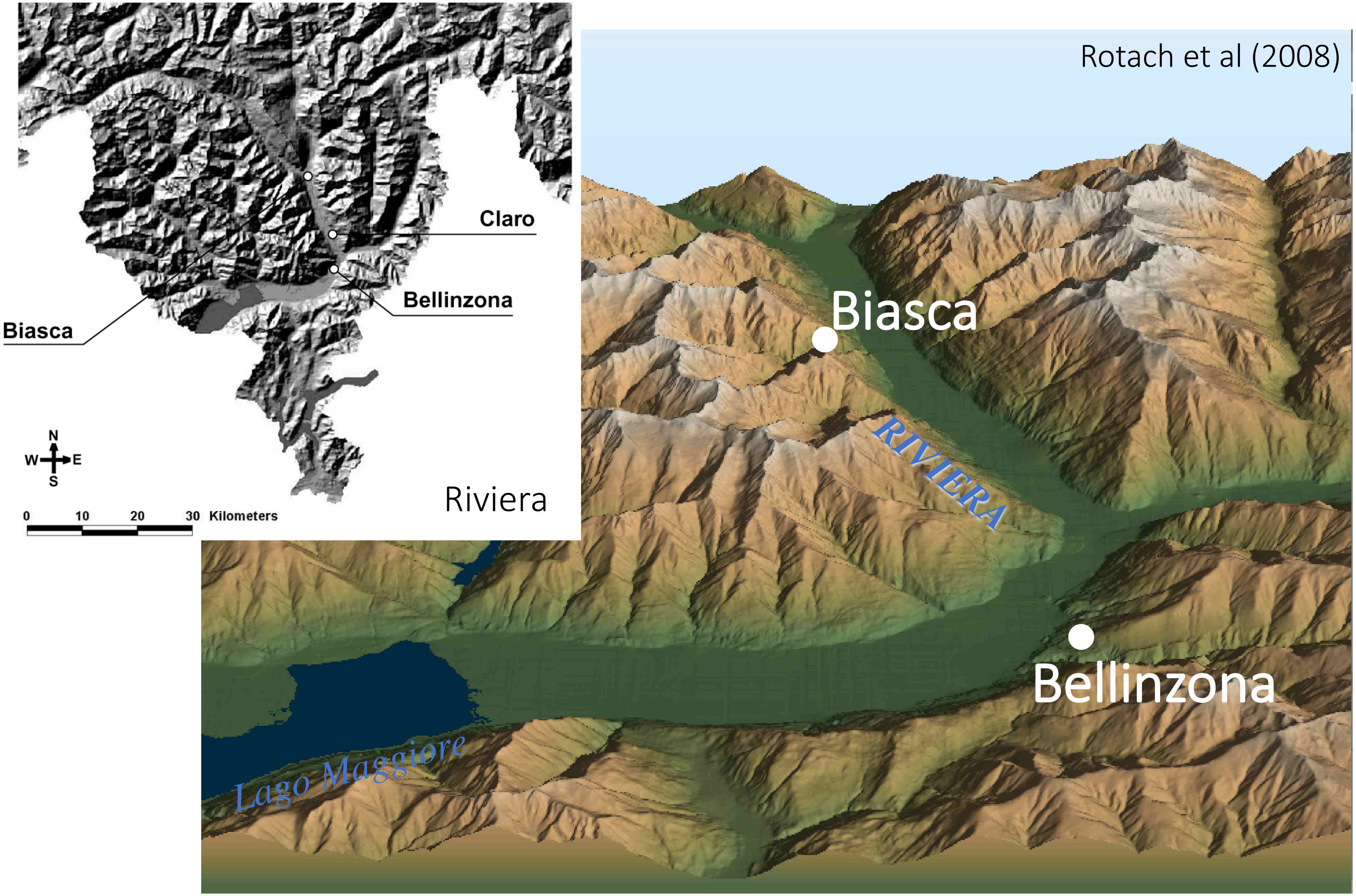
Buoyancy parameter: $g / \overline{\theta}$

MAP Riviera Project

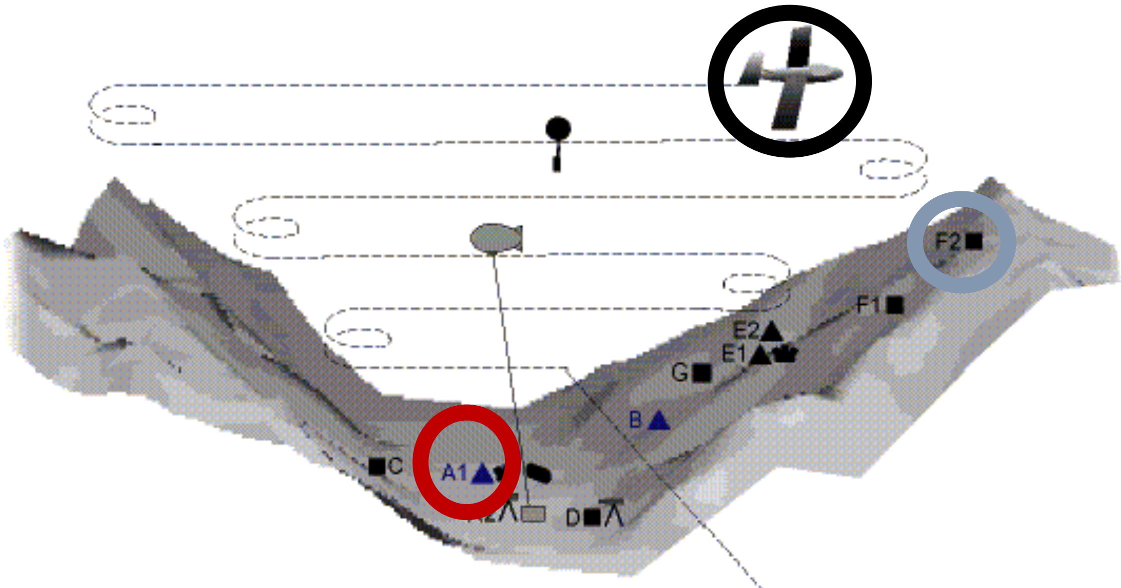
Rotach et al (2004)



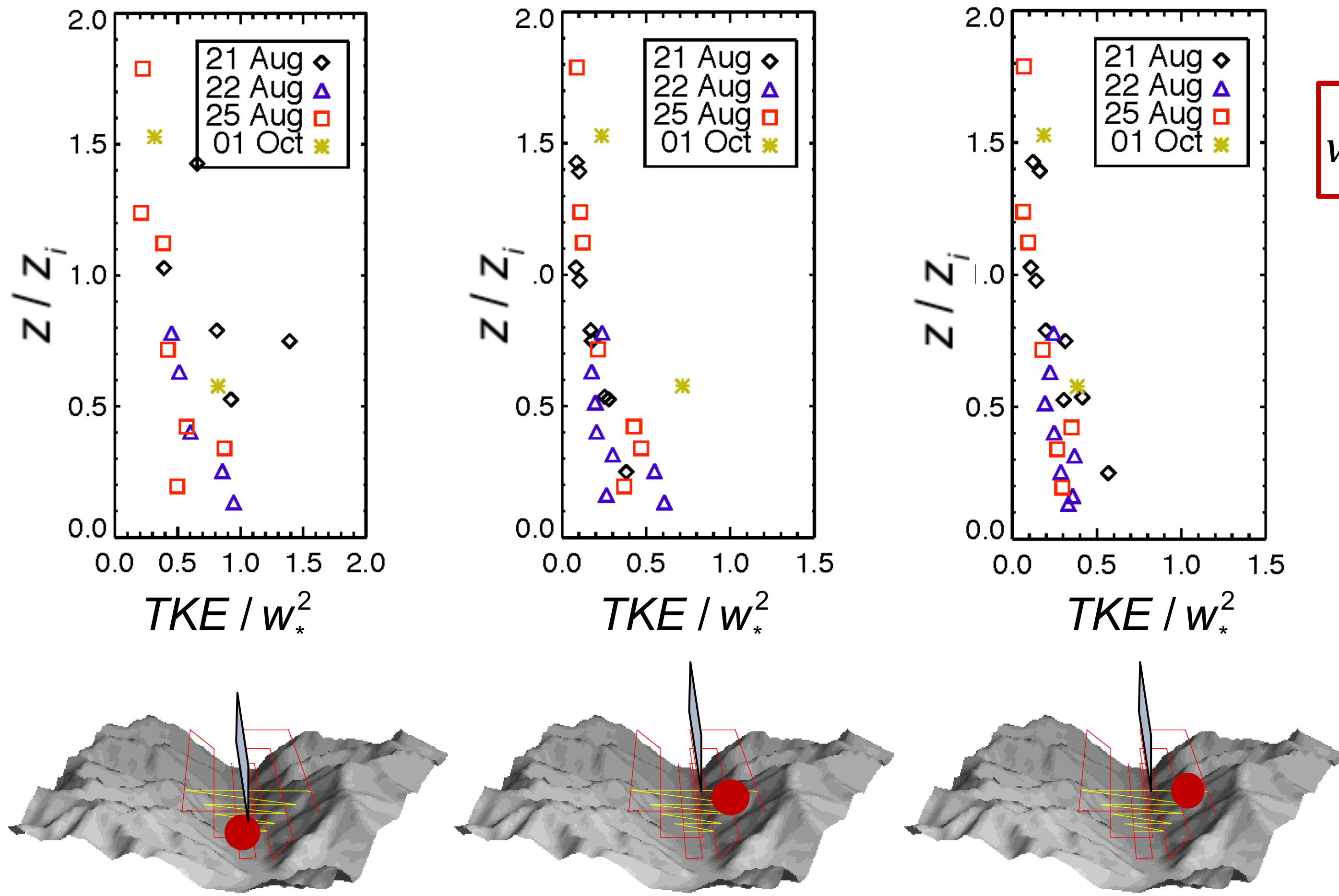
MAP Riviera Project



MAP Riviera Project: summer/fall 1999



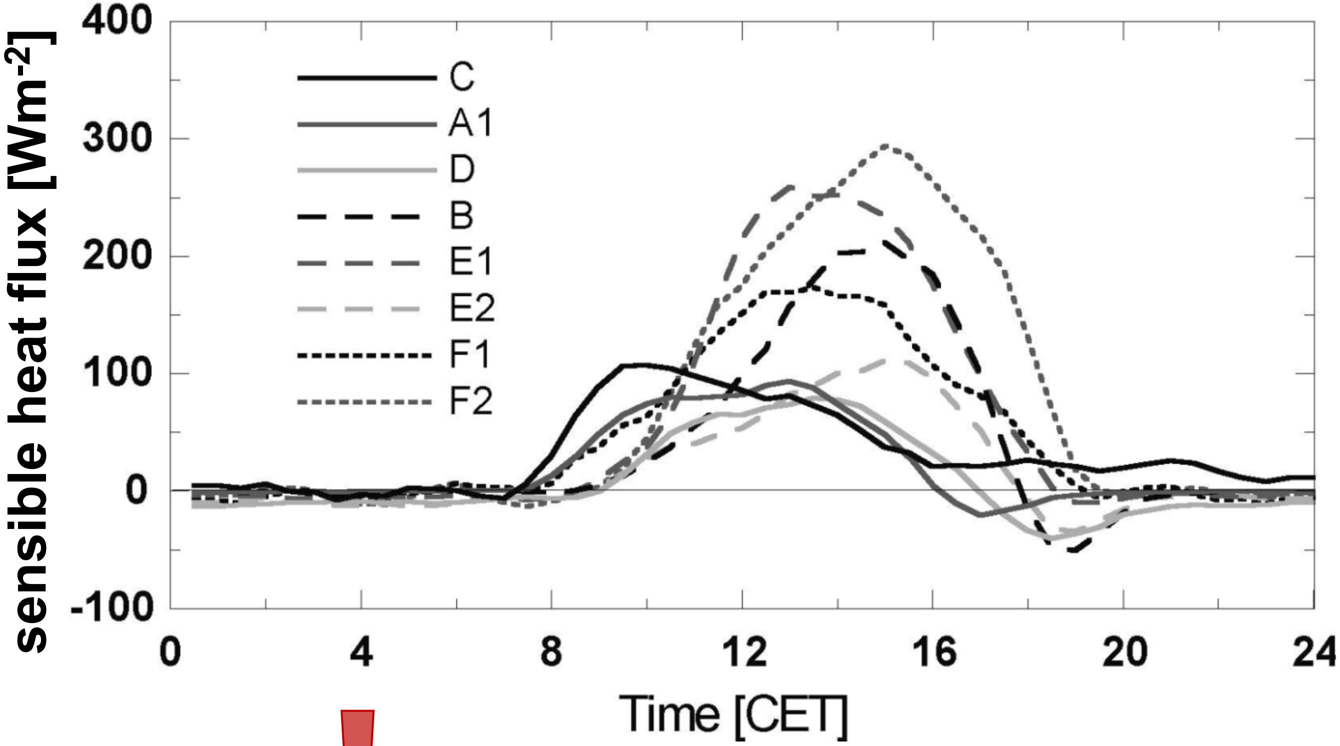
MAP: Measured Scaled TKE profiles

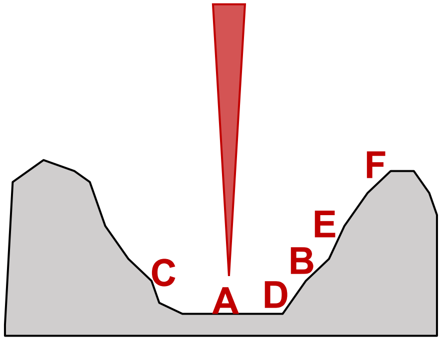


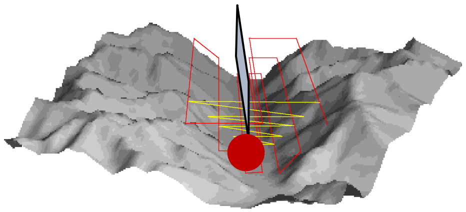
$$w_* = \left(\overline{w'\theta'}_o \frac{g}{\bar{\theta}} z_i\right)^{1/3}$$

Weigel and Rotach (2004)

MAP: Measured surface fluxes

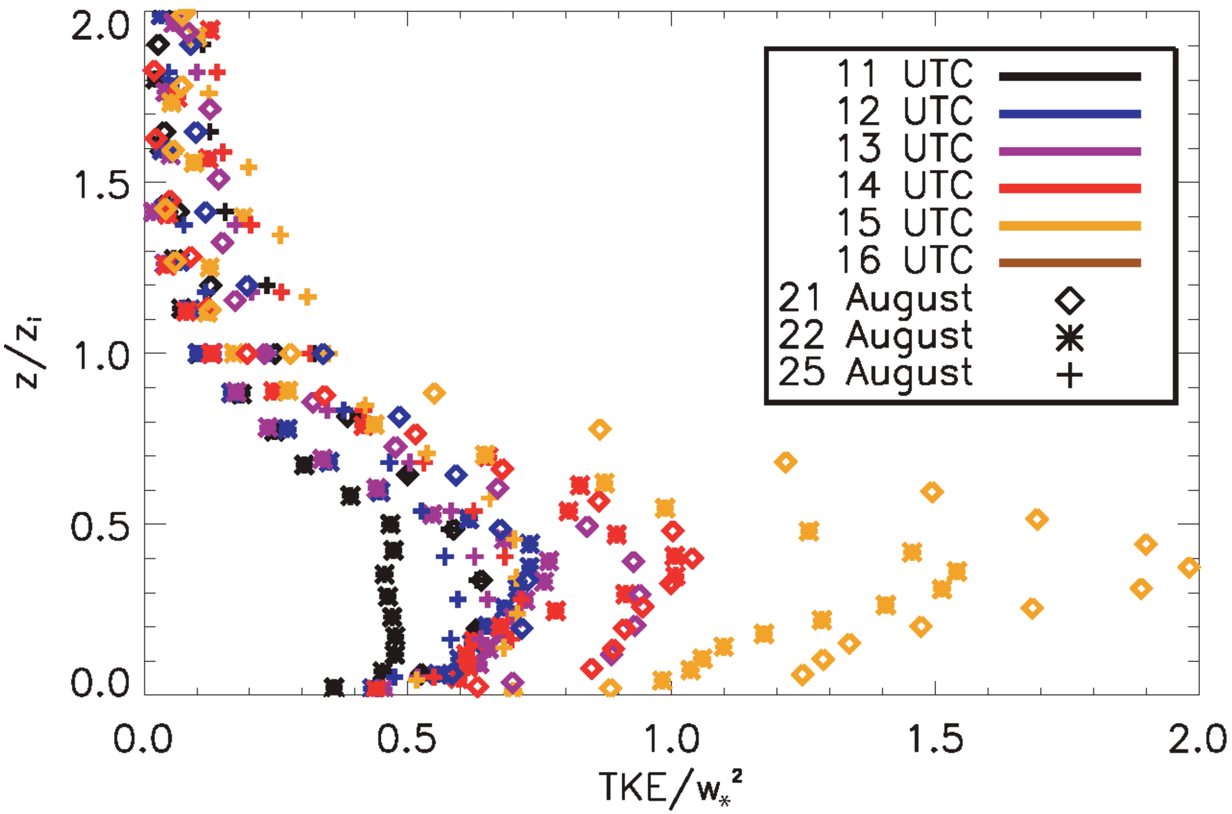


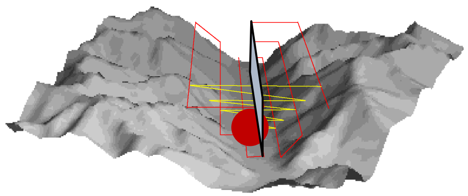




Rotach et al (2008)

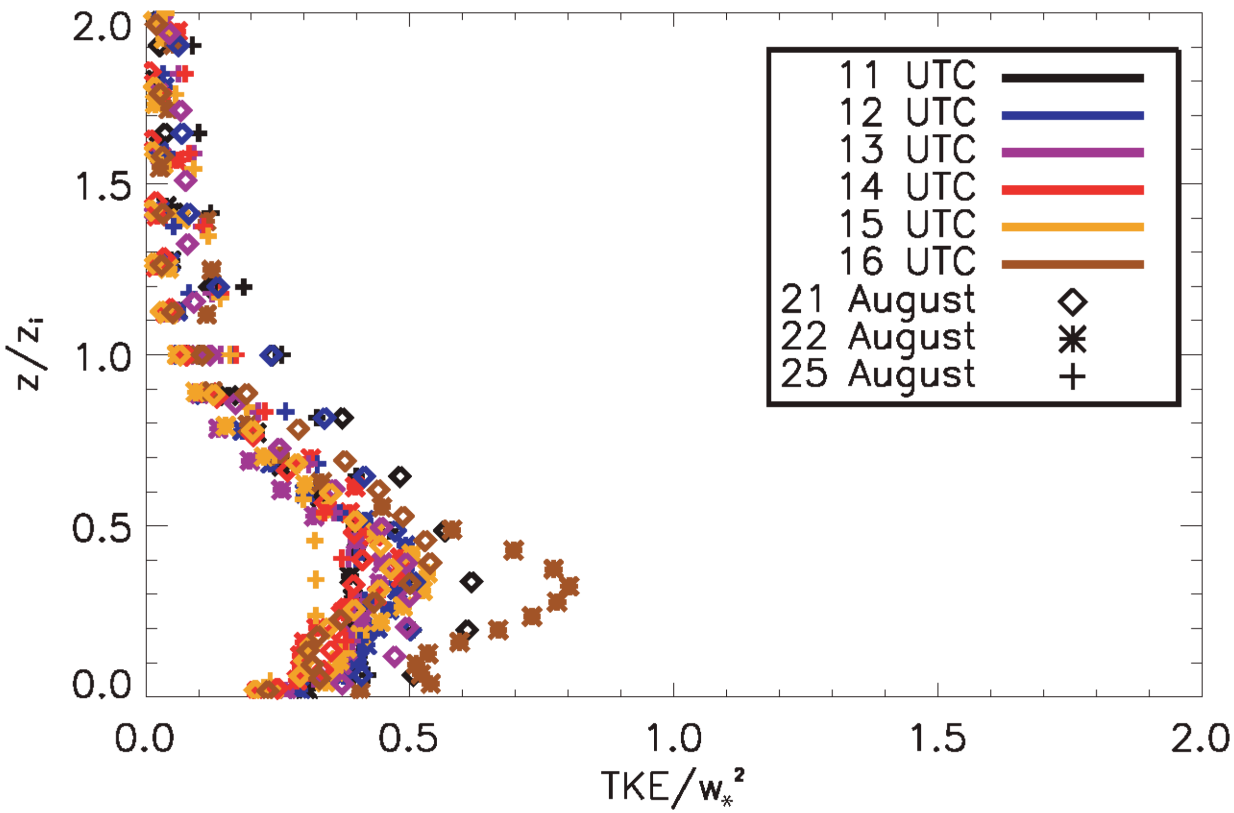
MAP: Simulated Scaled TKE profiles

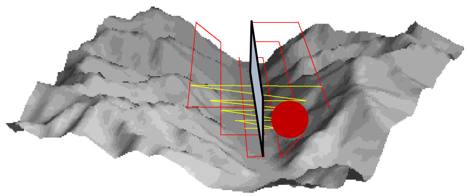




Weigel et al (BLM, 2007)

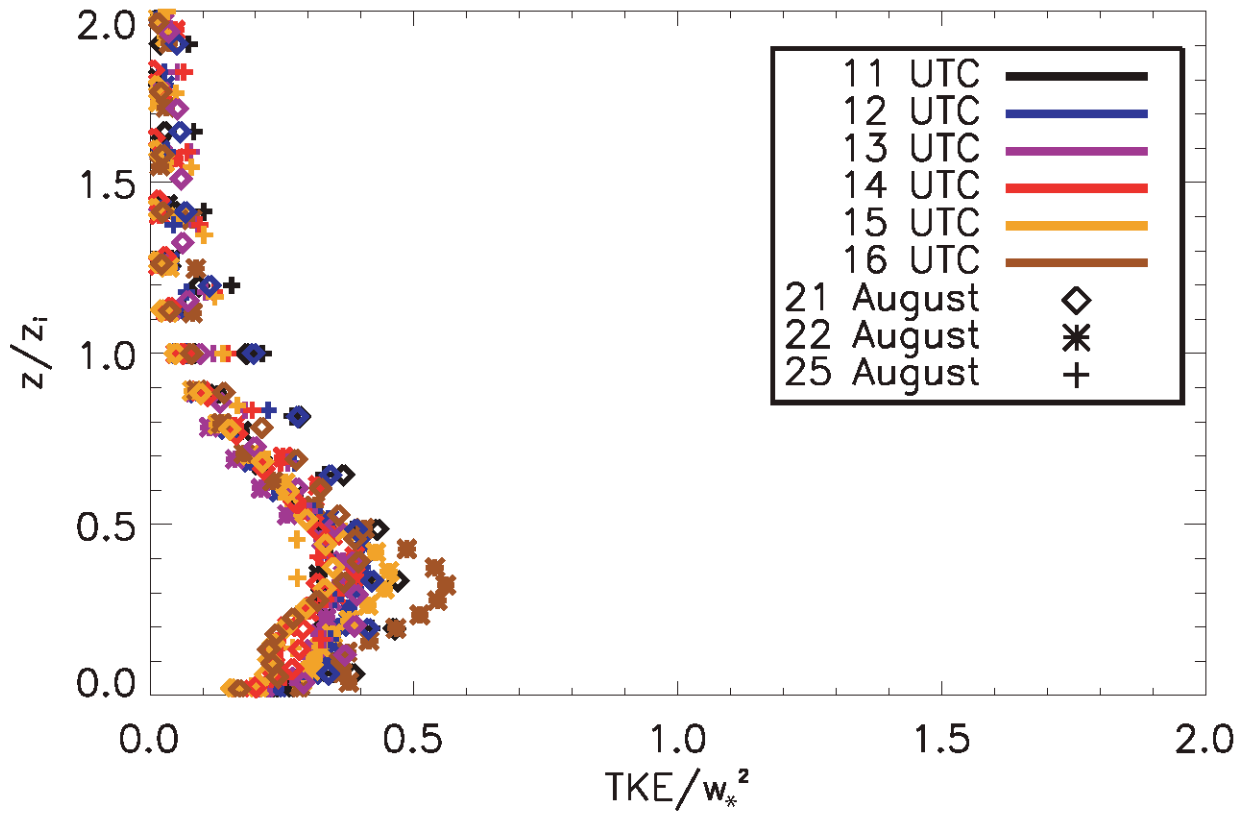
MAP: Simulated Scaled TKE profiles

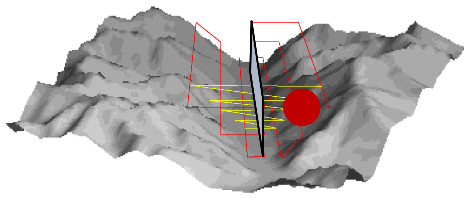




Weigel et al (BLM, 2007)

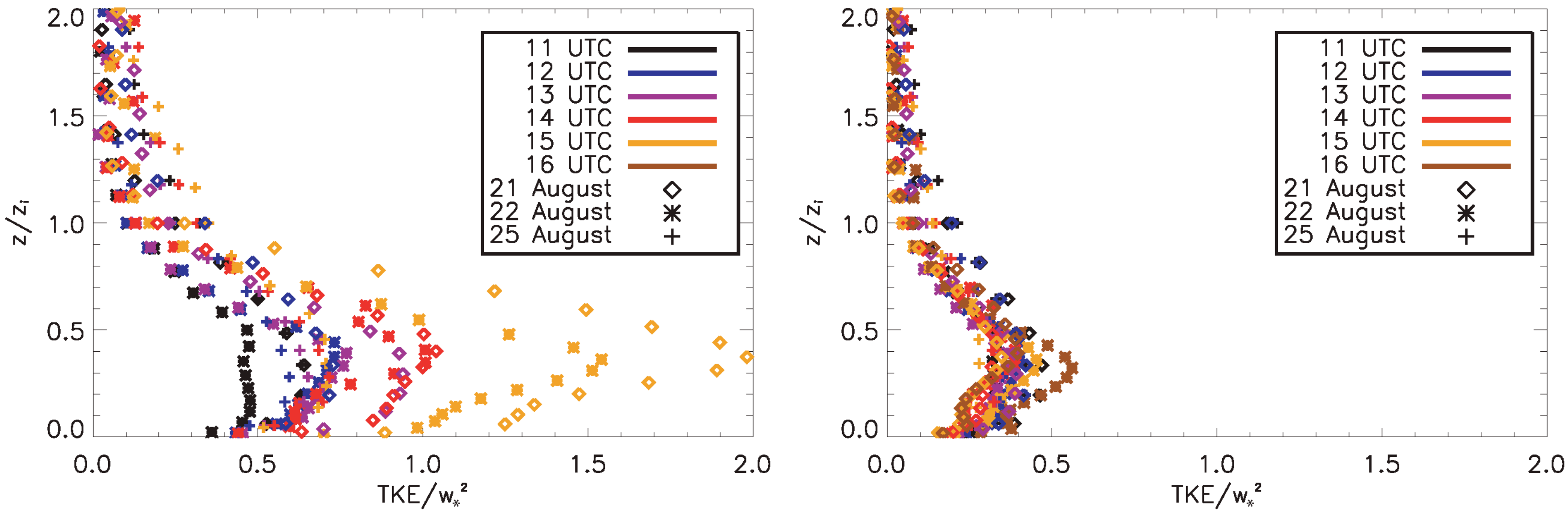
MAP: Simulated Scaled TKE profiles





Weigel et al (BLM, 2007)

MAP: Simulated Scaled TKE profiles



How can this scaling behavior be understood?
→ Look into simulated TKE budget

MAP: TKE production mechanisms

TKE budget

(solved in ARPS's 1.5 order TKE closure)

$$\frac{\partial \overline{e}}{\partial t} = \underbrace{-\overline{u}\left(\frac{\partial \overline{e}}{\partial x}\right) - \overline{v}\left(\frac{\partial \overline{e}}{\partial y}\right) - \overline{w}\left(\frac{\partial \overline{e}}{\partial z}\right)}_{\text{ADVECTION}} \underbrace{- \left(\overline{u'w'}\right)\left(\frac{\partial \overline{u}}{\partial z}\right) - \left(\overline{v'w'}\right)\left(\frac{\partial \overline{v}}{\partial z}\right)}_{\text{SHEAR PRODUCTION}} +$$

$$\underbrace{+ \frac{g}{\overline{\theta}}\left(\overline{w'\theta'}\right)}_{\text{BUOYANCY PRODUCTION}} \underbrace{- \frac{1}{\rho}\frac{\partial}{\partial z}\left(\overline{w'p'}\right) - \frac{\partial}{\partial z}\left(\overline{ew'}\right) - \epsilon}_{\text{PRESSURE CORRELATION, TURB. TRANSPORT, DISSIPATION}} .$$

ADVECTION

SHEAR PRODUCTION

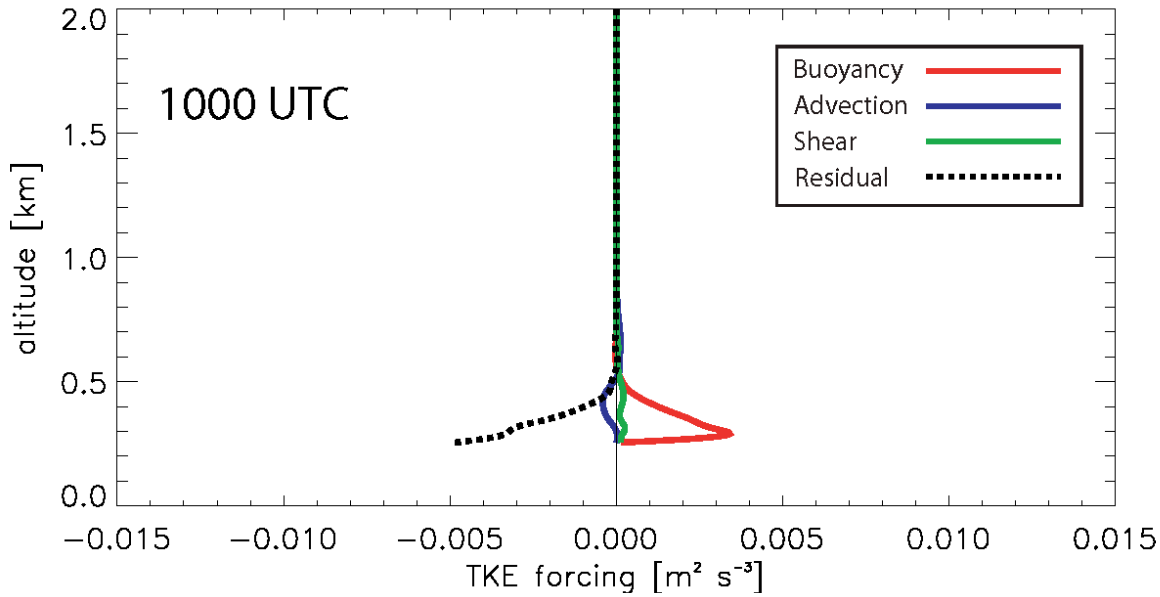
BUOYANCY PRODUCTION

PRESSURE CORRELATION, TURB. TRANSPORT, DISSIPATION

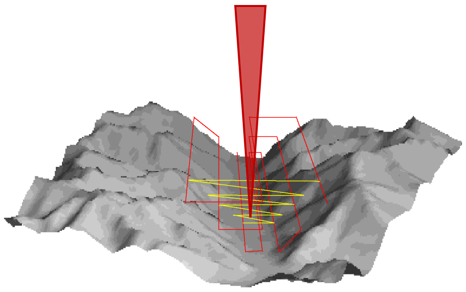
MAP: TKE production mechanisms

Evaluation of the TKE budget equation with ARPS

Late morning



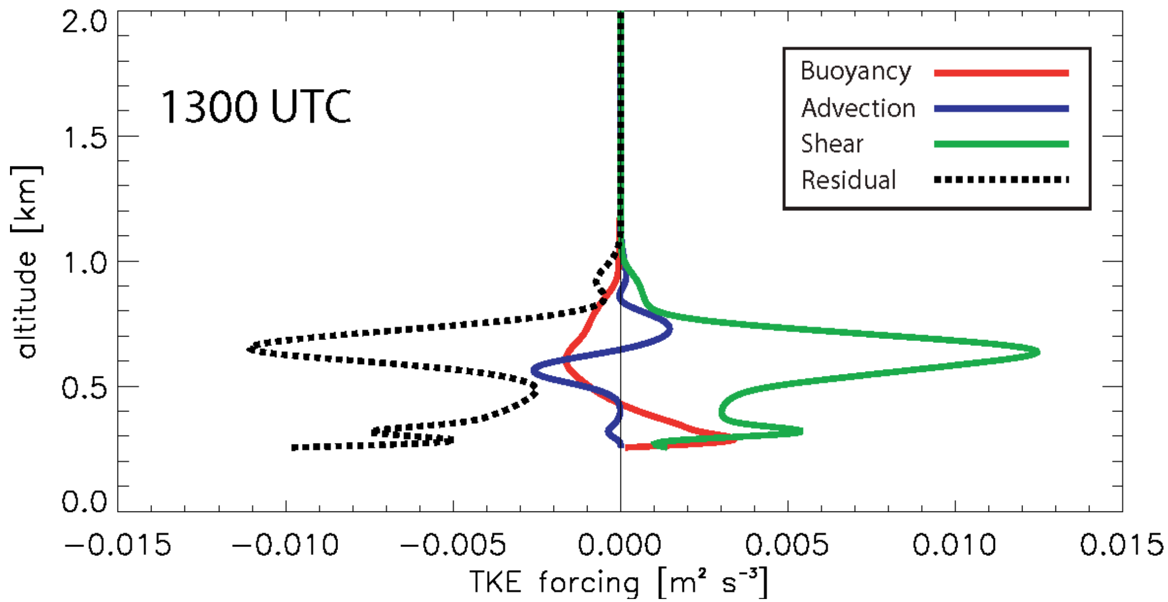
Example 22 August



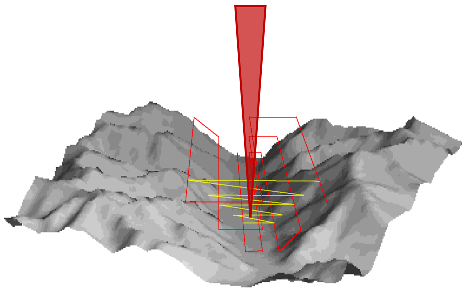
MAP: TKE production mechanisms

Evaluation of the TKE budget equation with ARPS

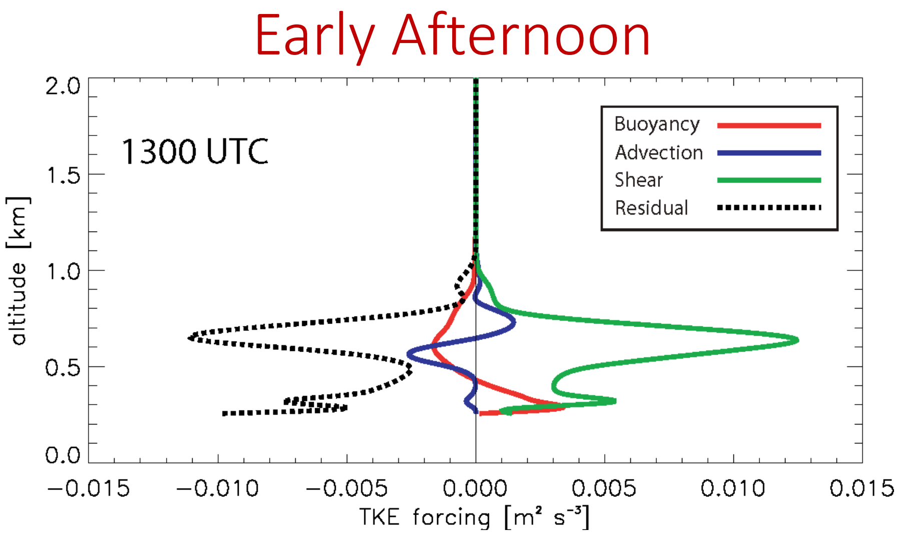
Early Afternoon

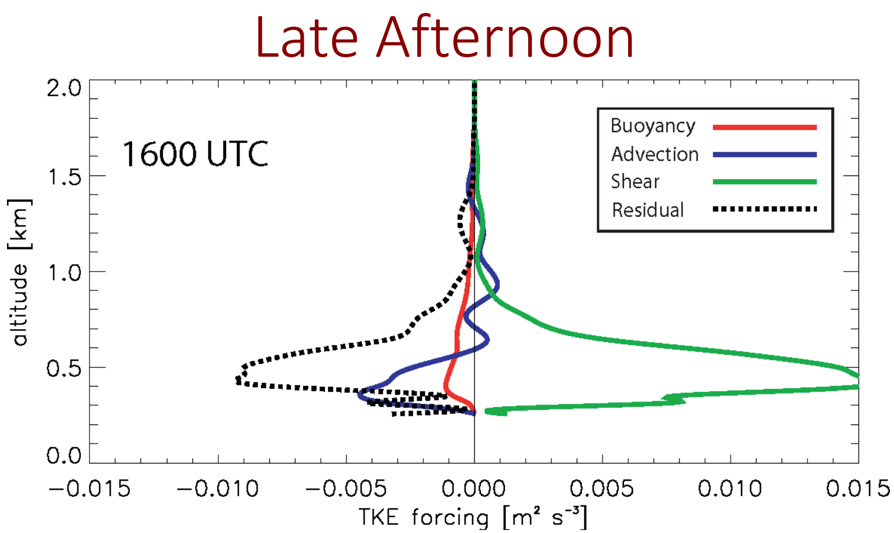


Example 22 August



MAP: TKE production mechanisms





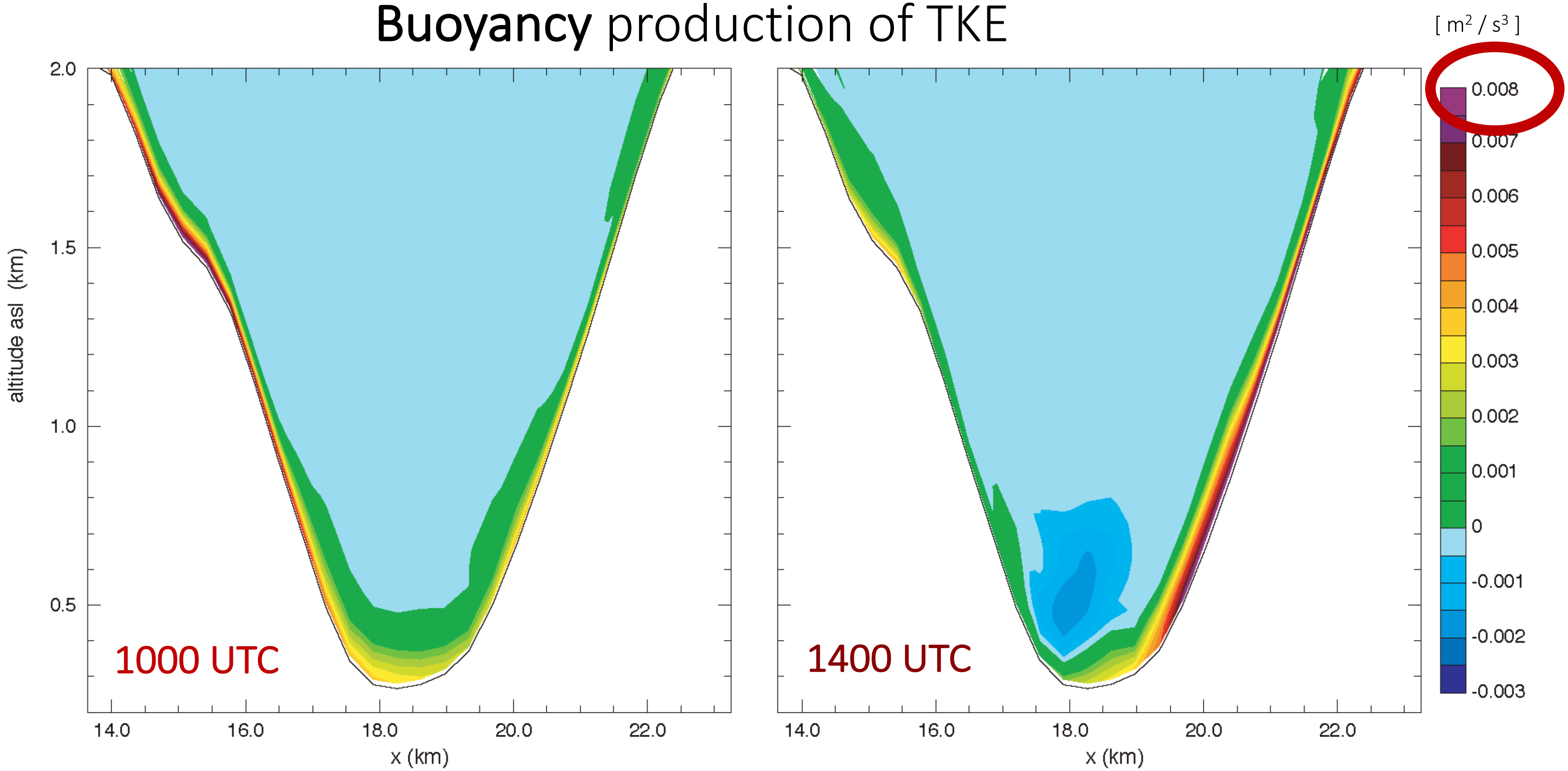
- Shear is dominant production mechanism

Turbulence is determined:

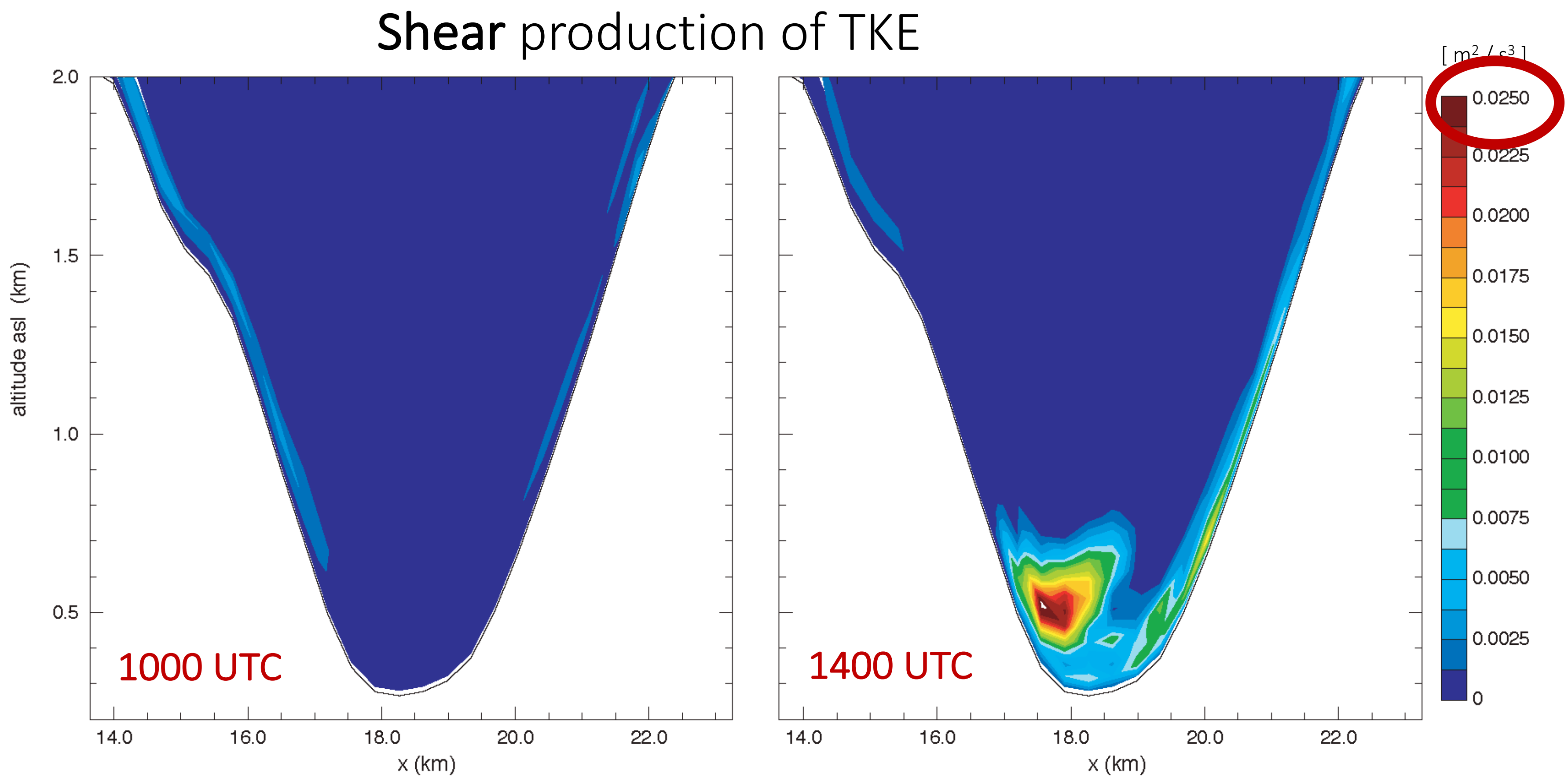
- by interaction of thermally driven valley flows
- not by buoyant processes from heated surface

MAP: TKE production mechanisms

Buoyancy production of TKE

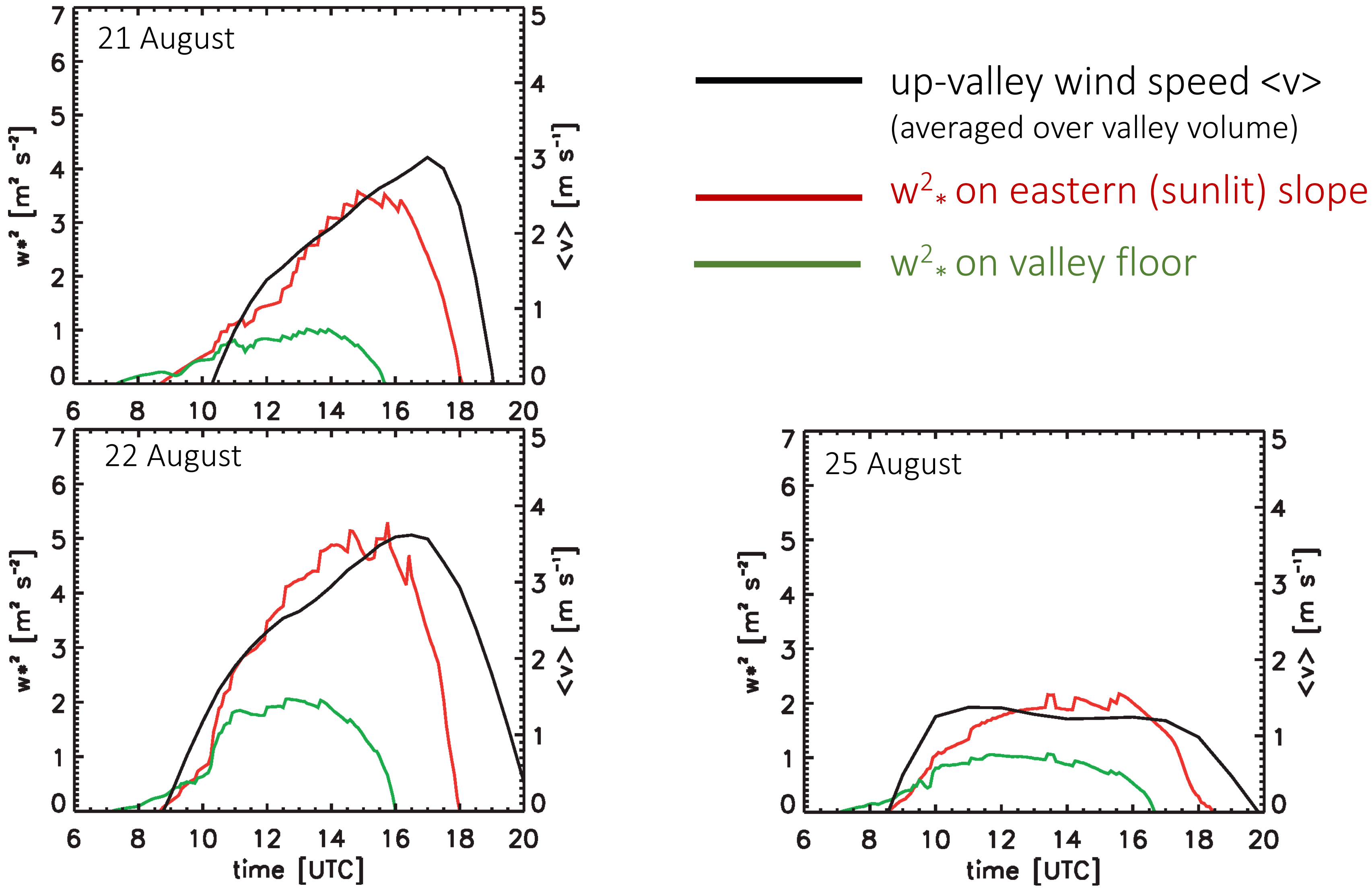


MAP: TKE production mechanisms



- TKE determined by strength of valley winds.
- But why does convective scaling approach still work ?

MAP: TKE production mechanisms

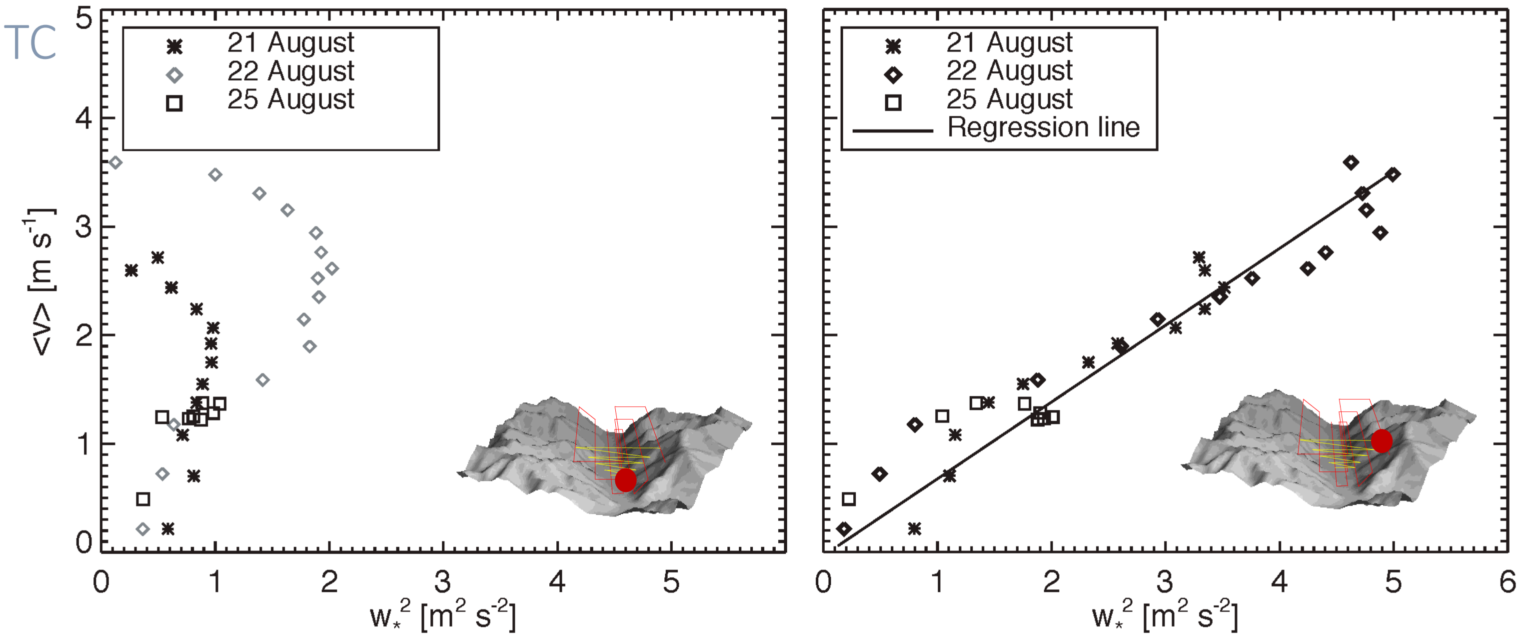


MAP: TKE production mechanisms

Correlation between **<v>** and **w^2_*** **$<v> \sim w^2_*$**

w^2_* from **valley floor** w^2_* from **sunlit slope**

9-16 UTC



- Coincidence?
- Problem with scaling: <v> has wrong dimensions...

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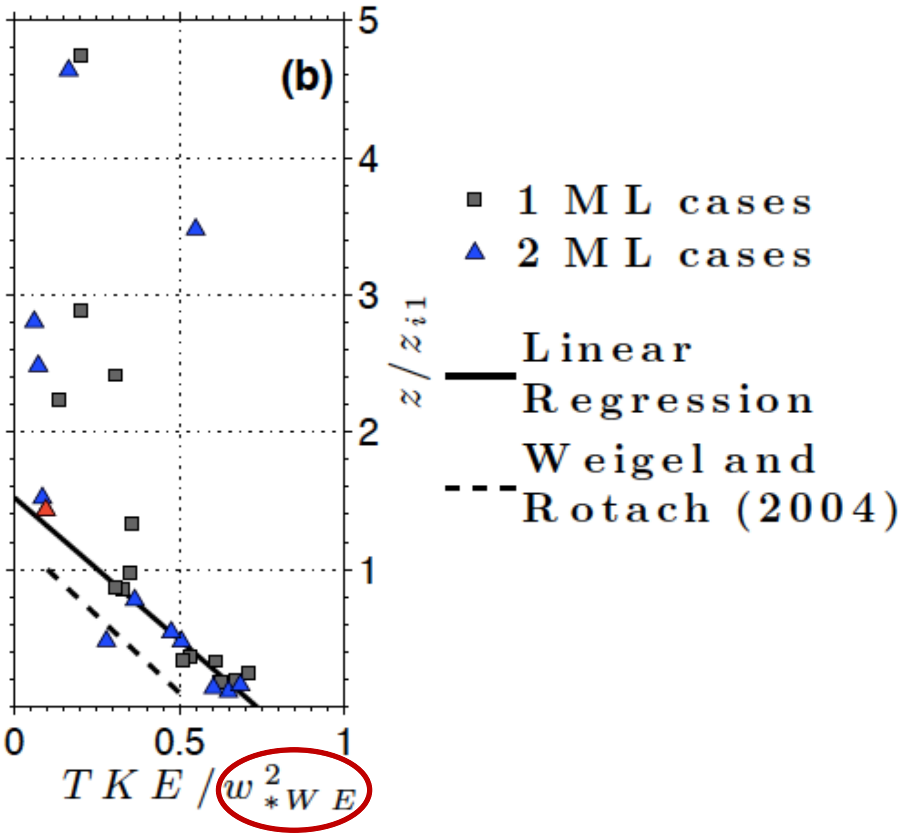
How general is this TKE scaling?

Inn Valley, 2013



→ east-west valley

→ also 'optimal' w_*

→ similar (but not equal) behaviour

Baur, MSc thesis (2015)

Scaling in complex terrain

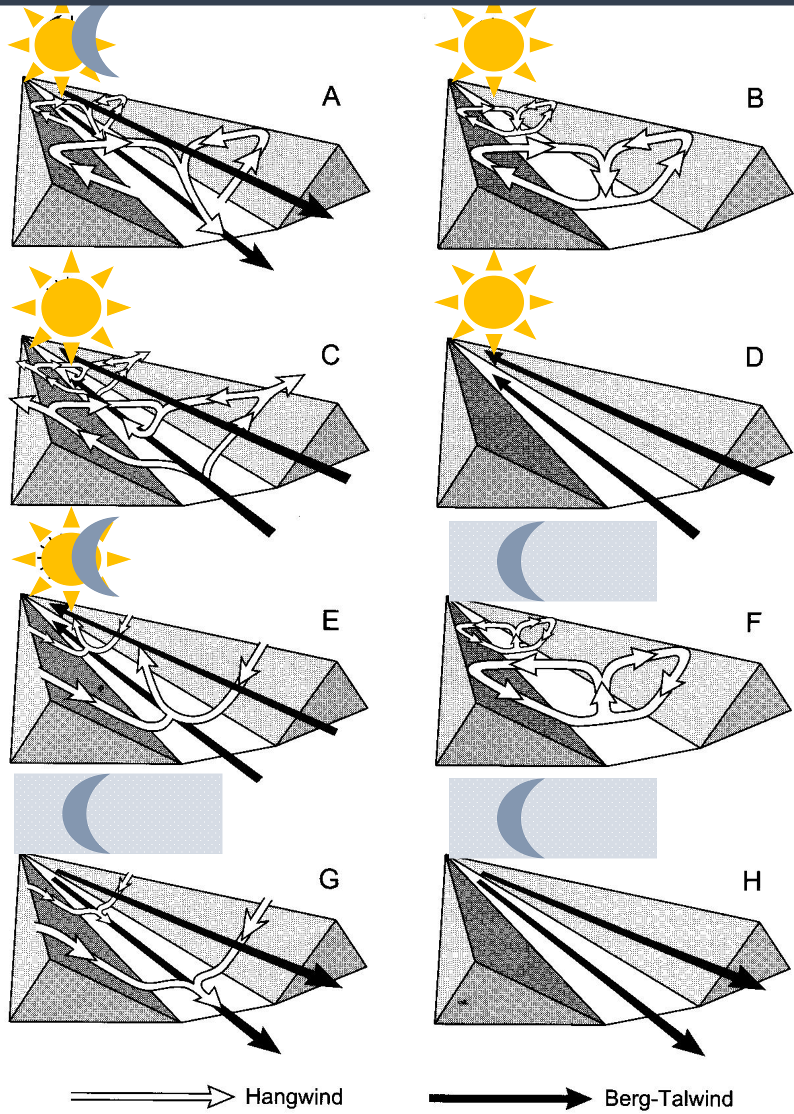
Application Examples:

1. Mixed-layer TKE scaling (MAP Riviera)
2. Wind directional change with height
3. Local scaling flux-variance relationships (12 Datasets)

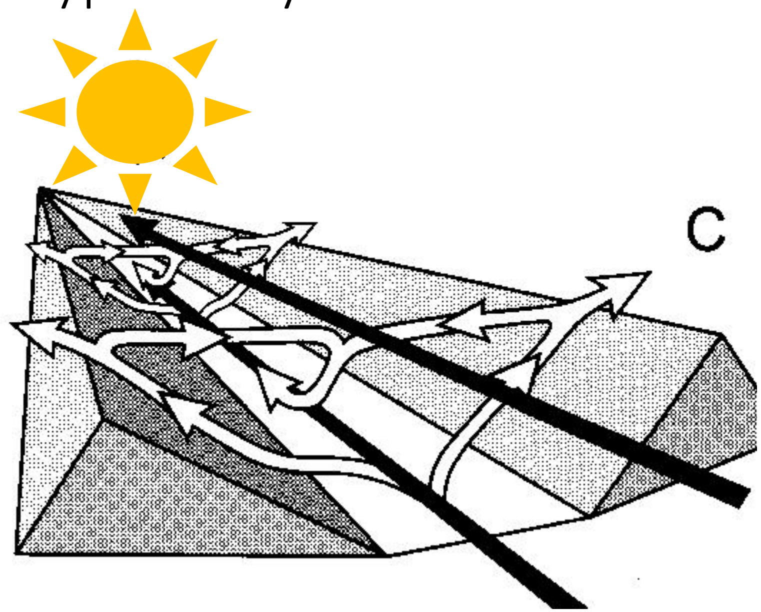
→ friction velocity $u_* =: \left[(\overline{u'w'_o})^2 + (\overline{v'w'_o})^2 \right]^{1/4}$

→ if coordinate system aligned with mean wind and no directional shear with height: $\overline{v'w'_o} = 0$

Valley wind – Slope wind

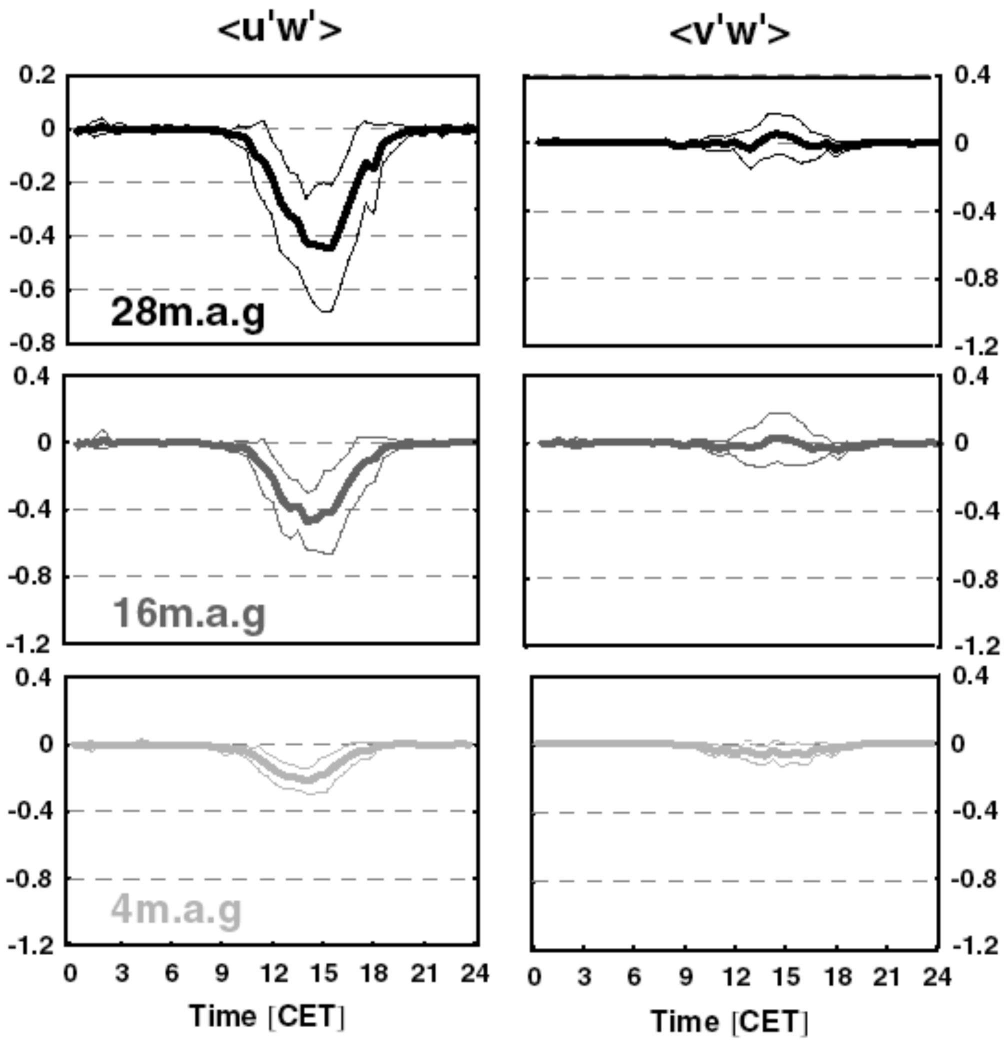


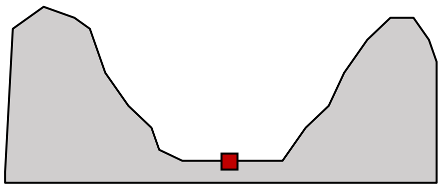
typical daytime



after Defant, 1949

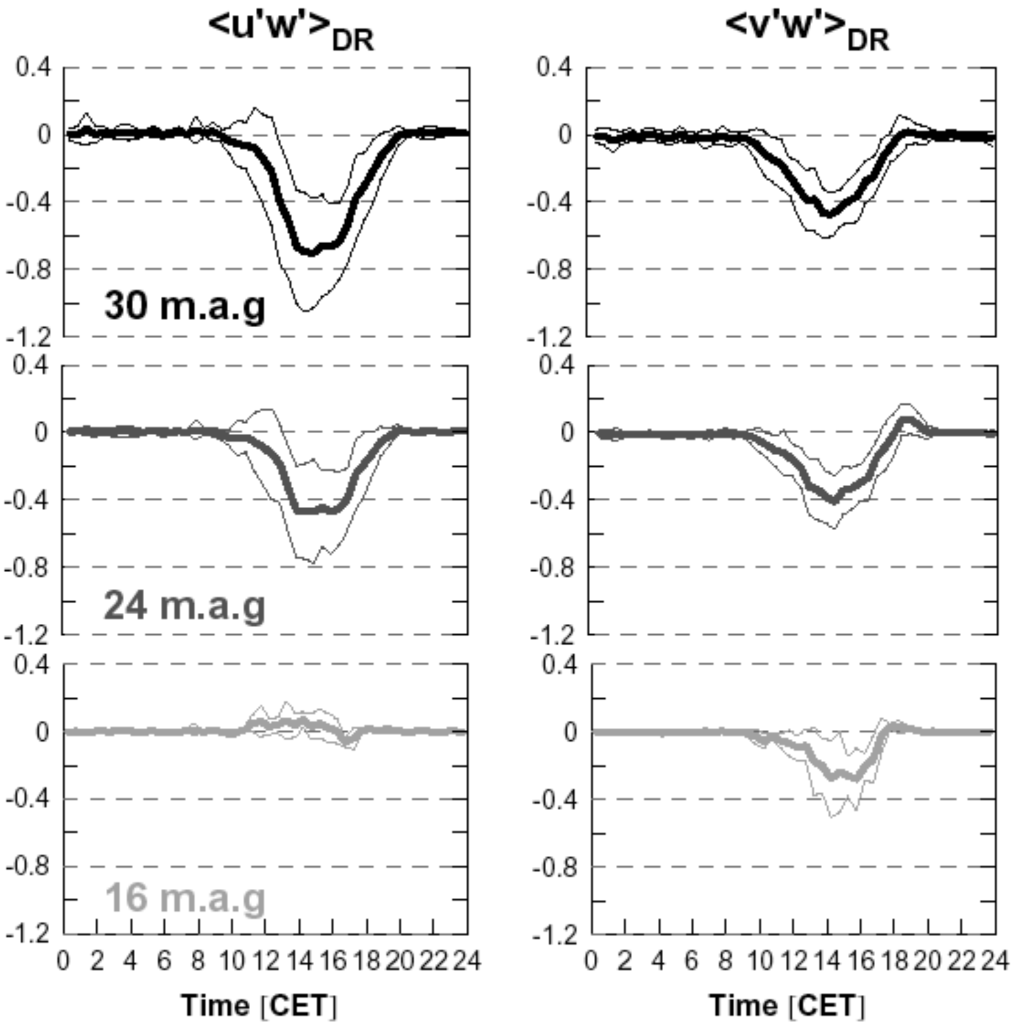
Momentum transport: Valley floor

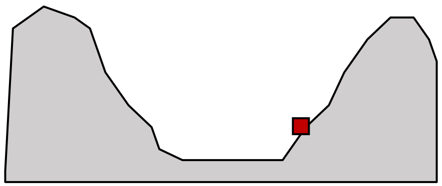




frictional stress: $\overline{u'w'}$

Momentum transport: Slope station





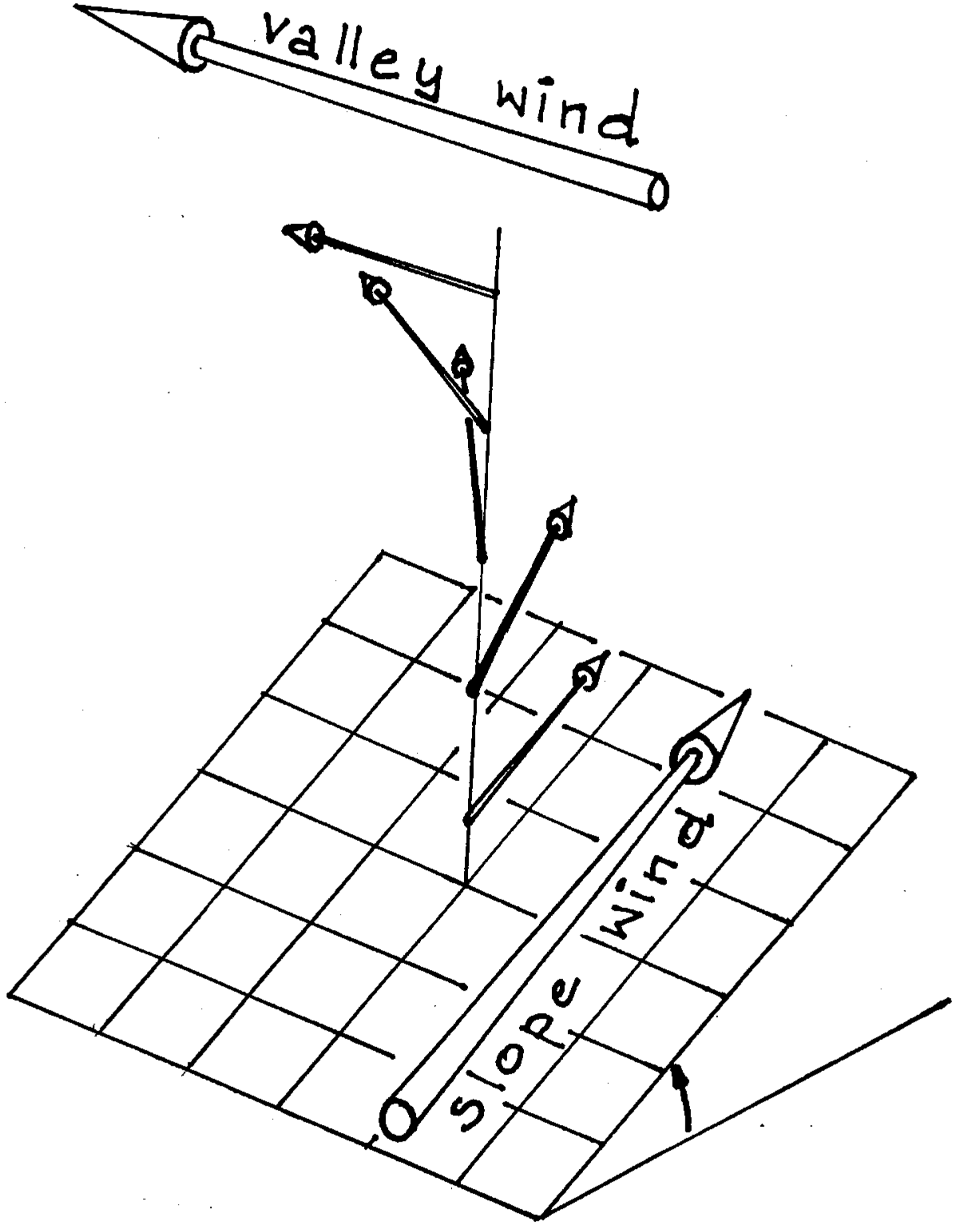
Local circulations cause :

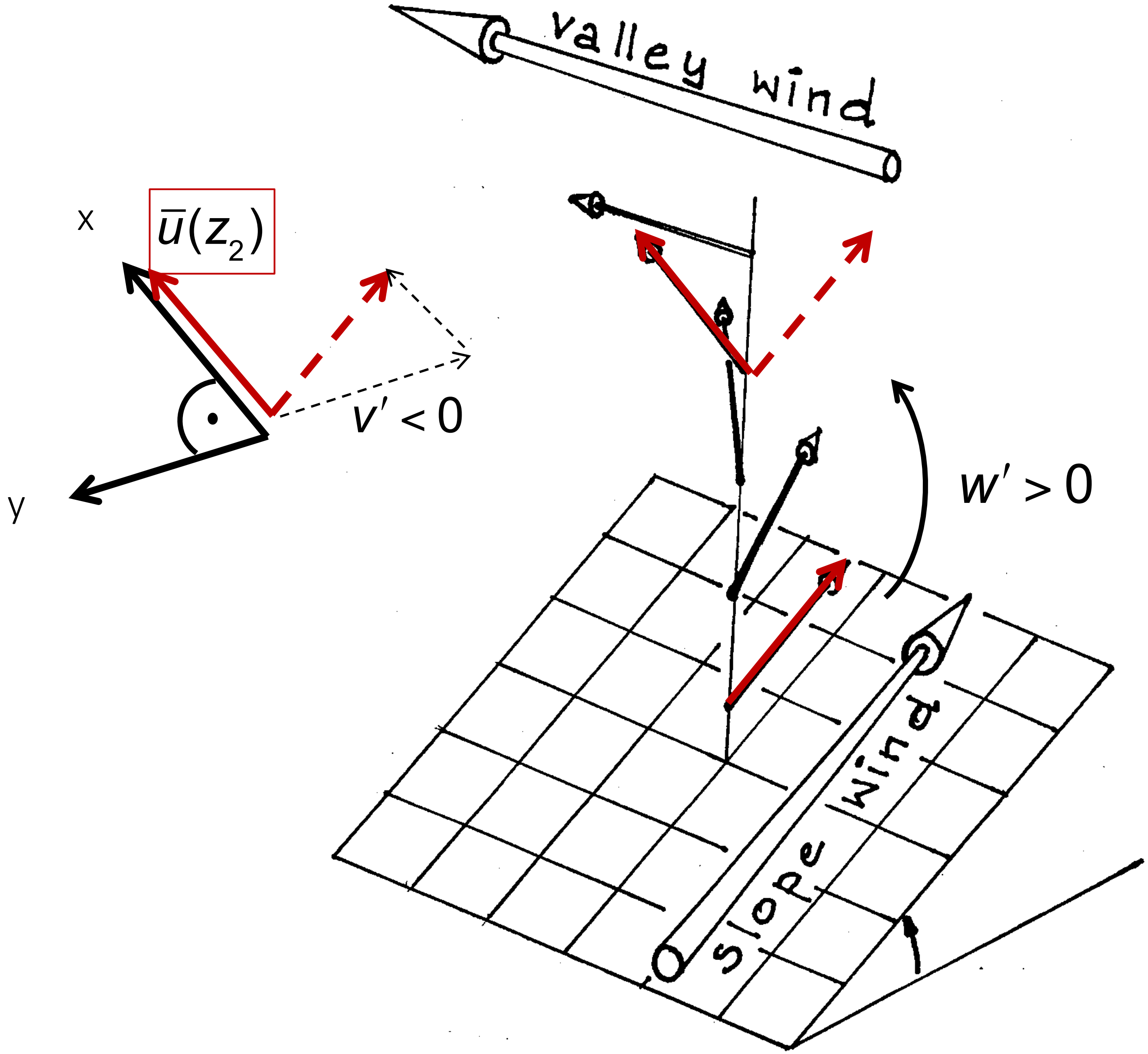
frictional stress: $\overline{u'w'}$

+

directional stress: $\overline{v'w'}$

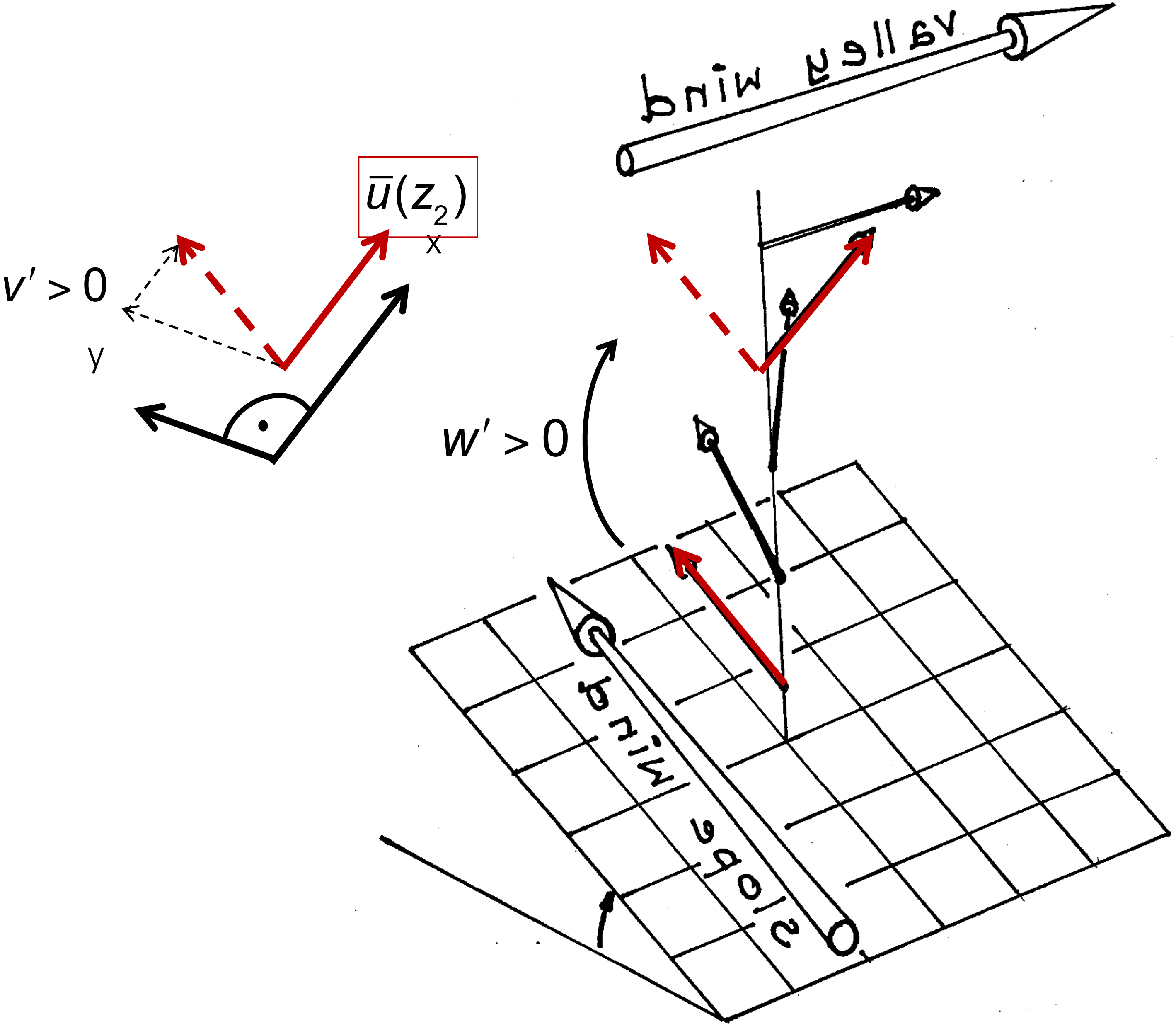
Andetta et al. (2001)


valley wind
slope wind



this eddy:
$v'w' < 0$

many eddies:
$\overline{v'w'} < 0$



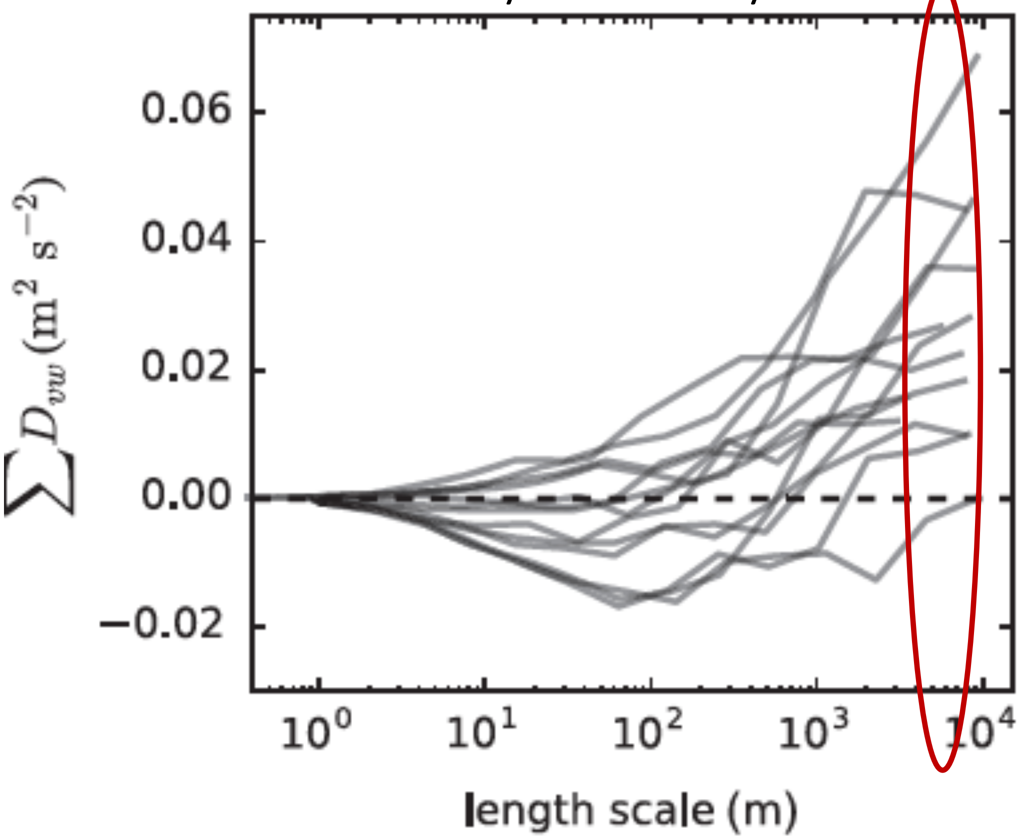
other slope:

$$\overline{v'w'} > 0$$

Momentum transport: Opposite slope

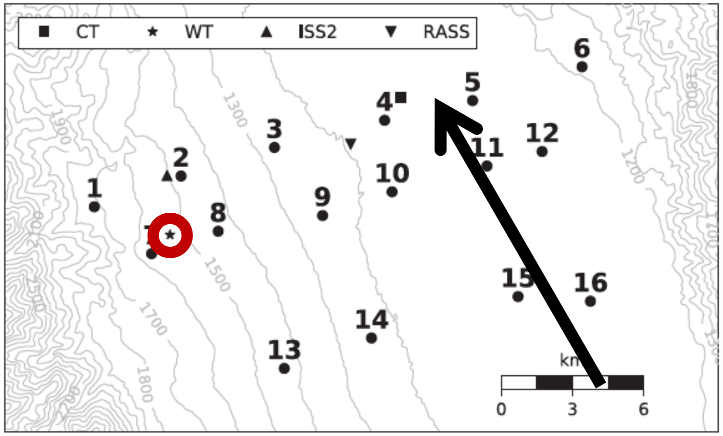
→ summed contributions to v'w'

valley wind days



→ <v'w'> all scales

Owens Valley, CA



Babic N et al. (2017)

Momentum transport: Scaling velocity

flat terrain:
→ in streamline coordinates

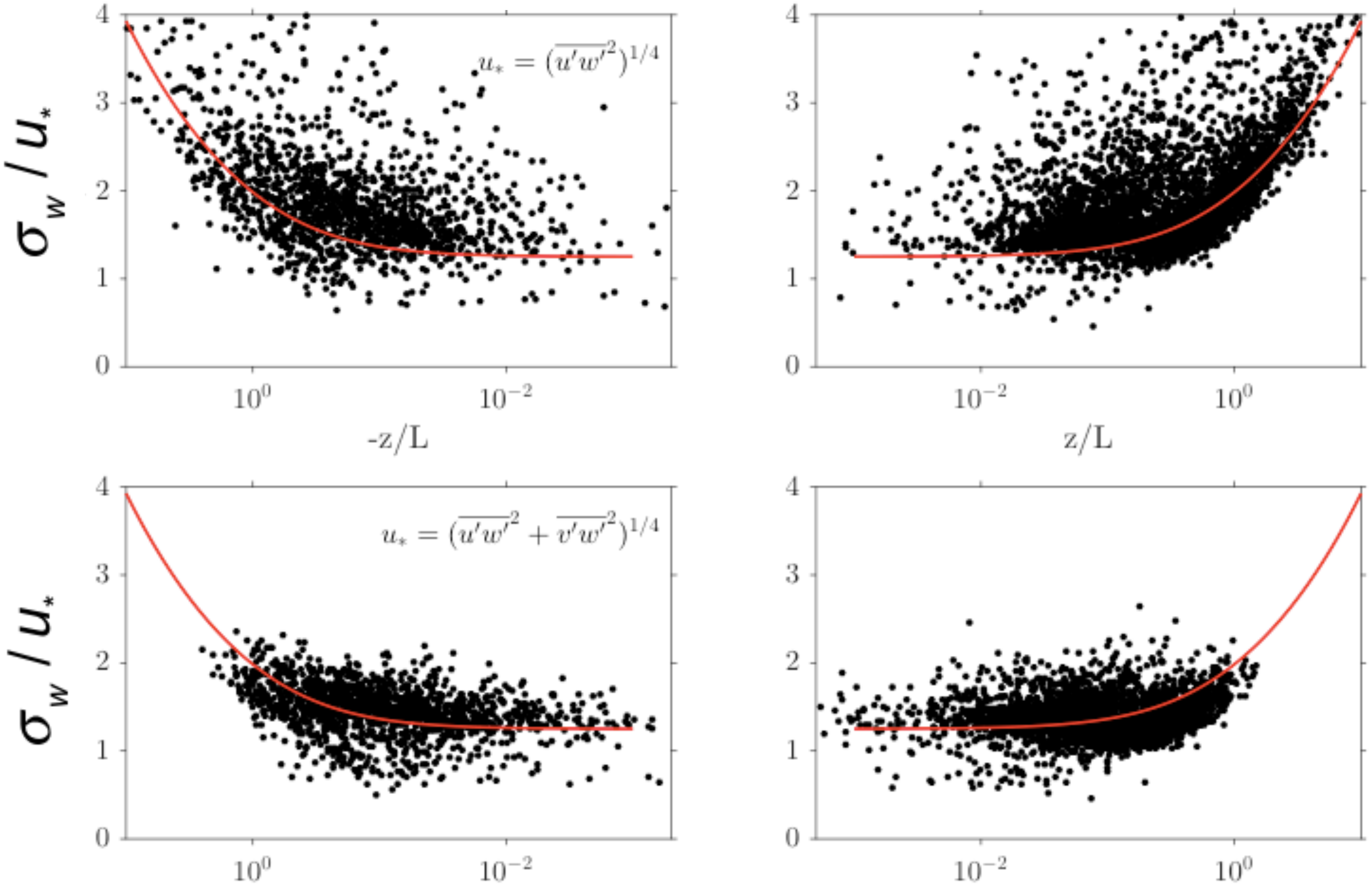
$$u_* = \left(-\overline{u'w'}\right)^{1/2}$$ is enough, because $$\overline{v'w'} = 0$$

systematic turning with height (e.g., slope/valley wind)
→ even if streamline coordinates

$$u_* = \left(\overline{u'w'}^2 + \overline{v'w'}^2\right)^{1/4}$$ necessary

Momentum transport: Scaling velocity

unstable | stable



$$u_* = \left(-\overline{u'w'}^2\right)^{1/2}$$

$$u_* = \left(\overline{u'w'}^2 + \overline{v'w'}^2\right)^{1/4}$$

-z/L z/L

data from i-Box site ‚Hochhäuser', 2014-2016, HQ

Scaling in complex terrain

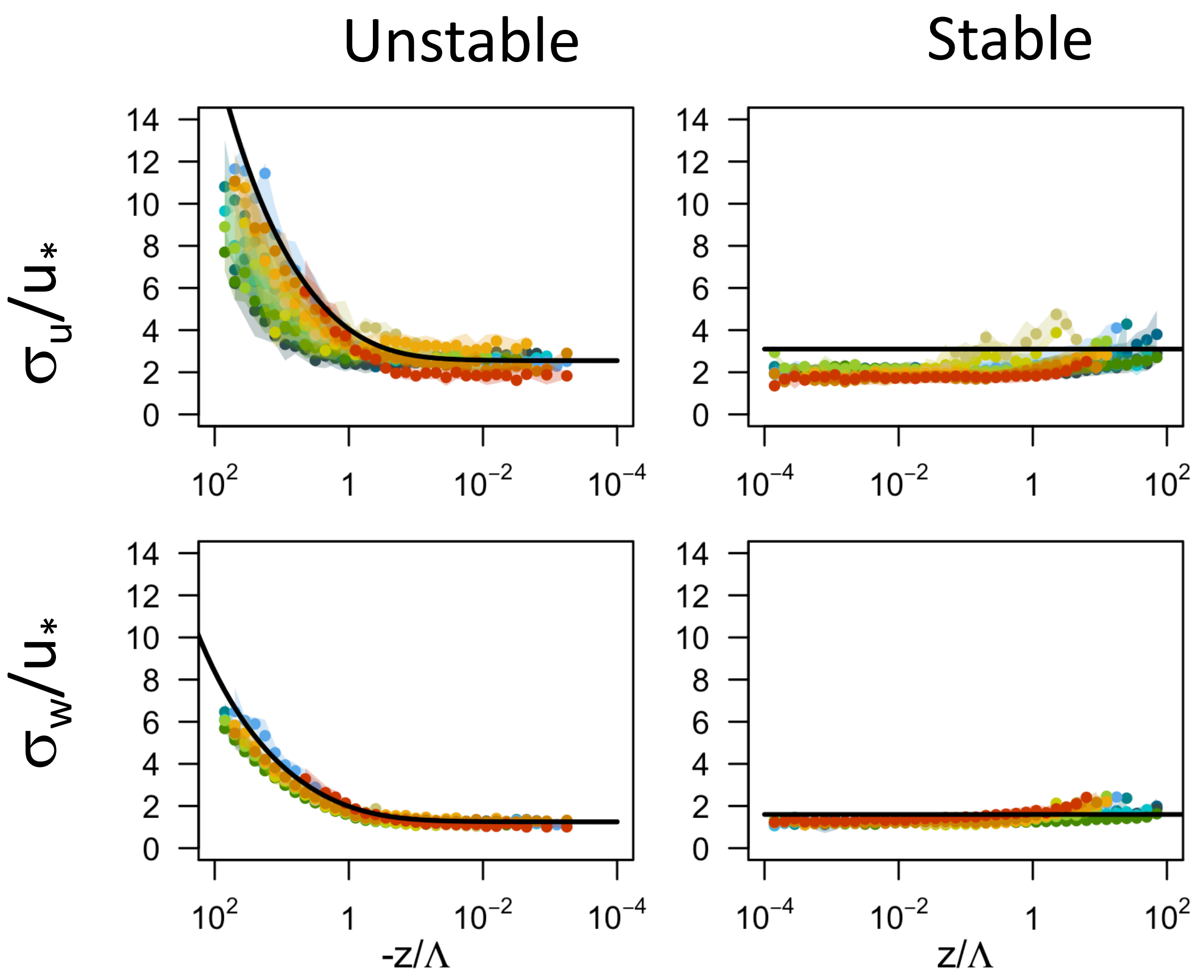
Application Examples:

1. Mixed-layer TKE scaling (MAP Riviera)
2. Wind directional change with height
3. Local scaling flux-variance relationships (12 Datasets)

→ look at influence of anisotropy

Scaling in complex terrain

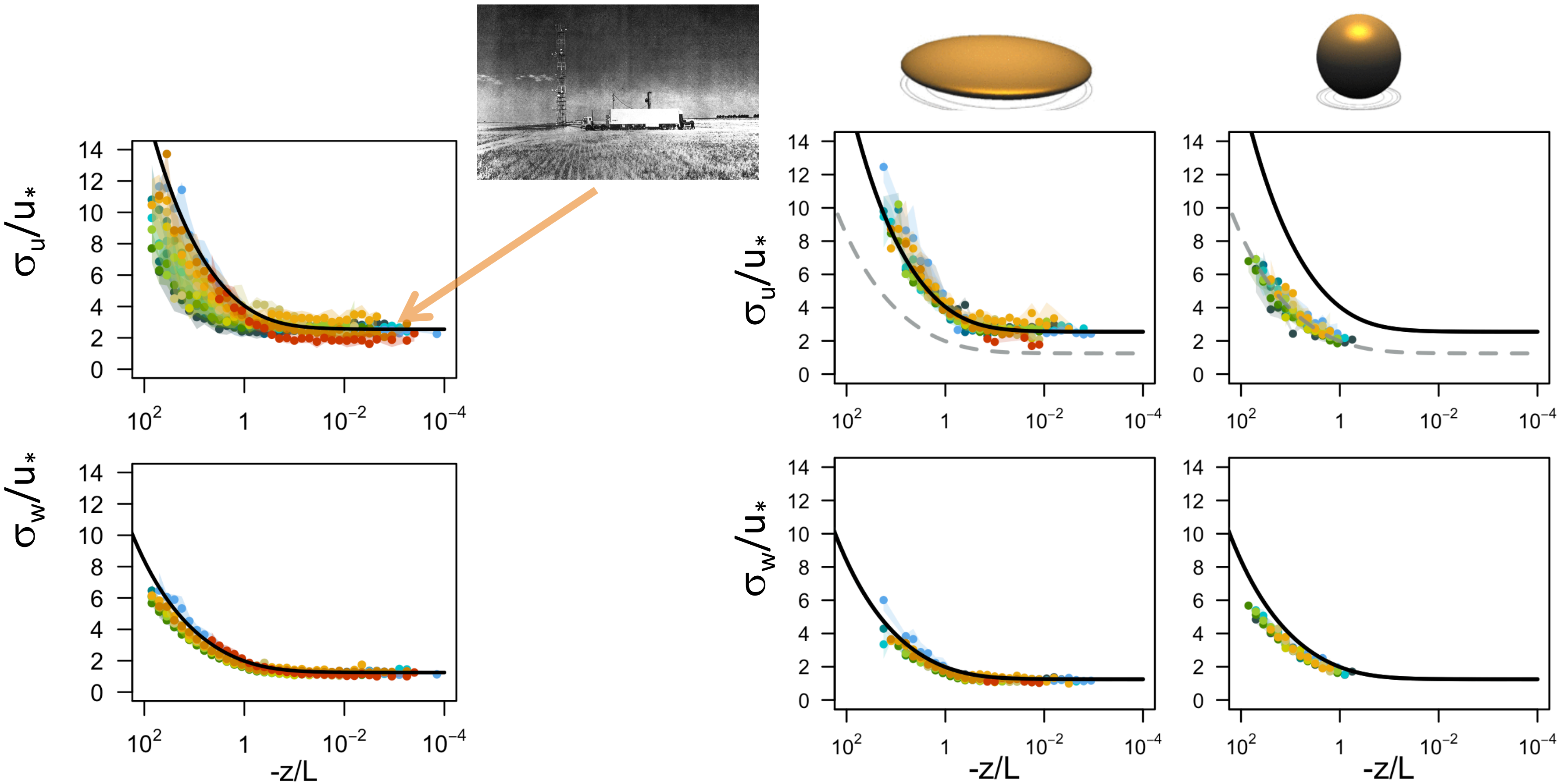
Standard deviation horizontal and vertical velocity



Stiperski et al. (2019)

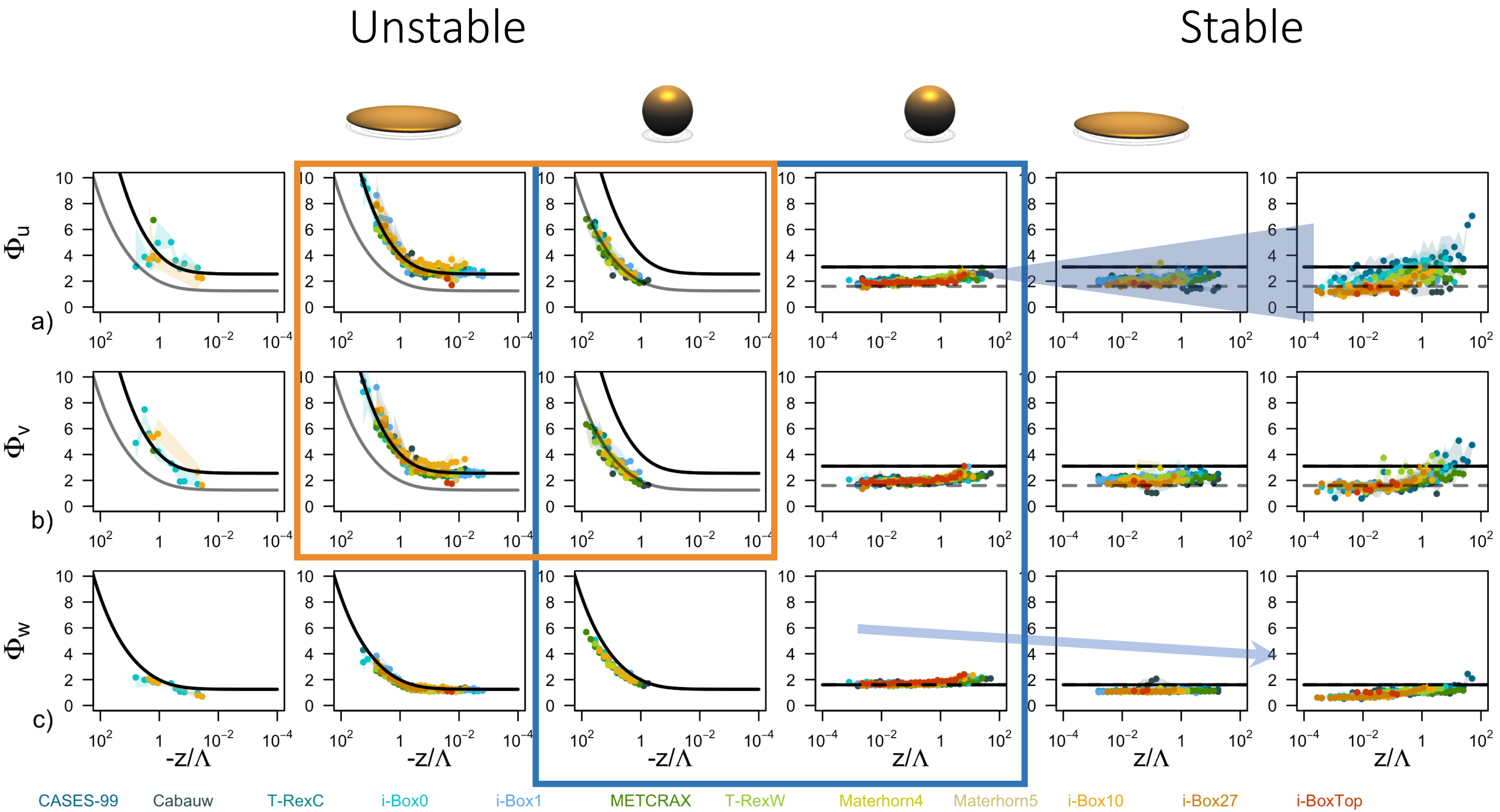
Scaling in complex terrain

Standard deviation horizontal and vertical velocity



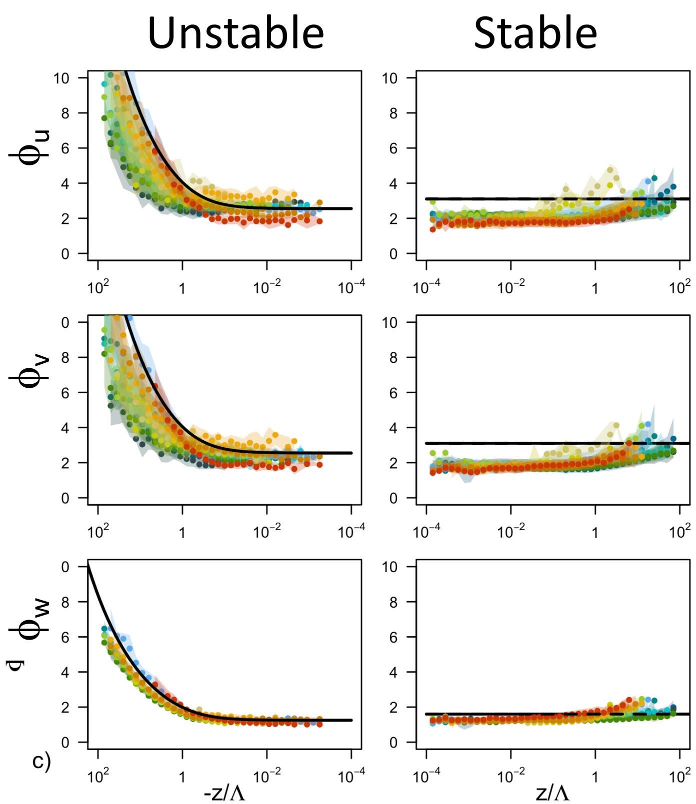
Stiperski et al. (2019)

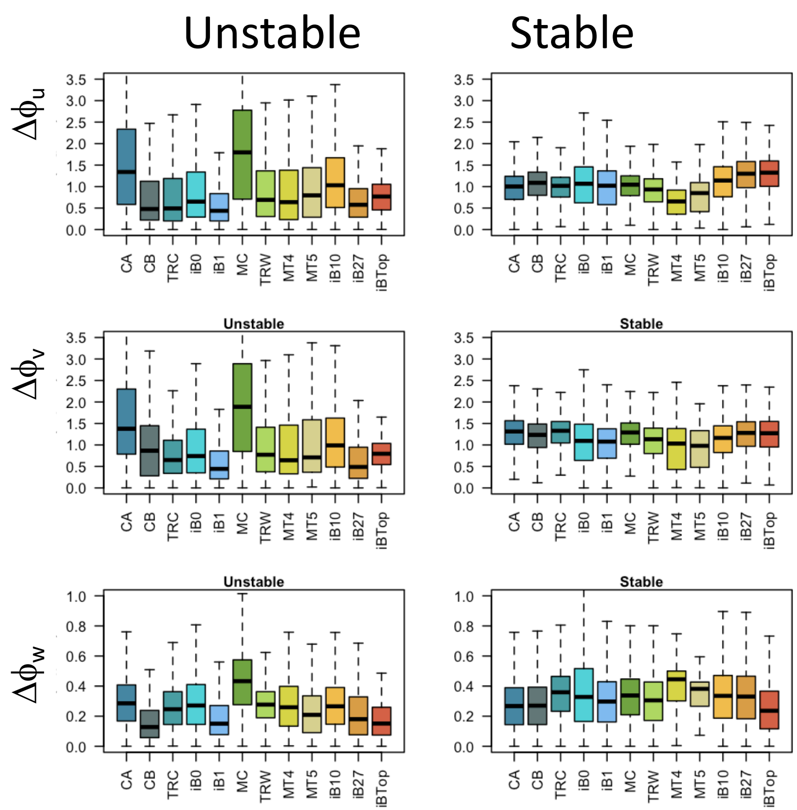
Scaling in complex terrain



Stiperski et al. (2019)

Scaling in complex terrain

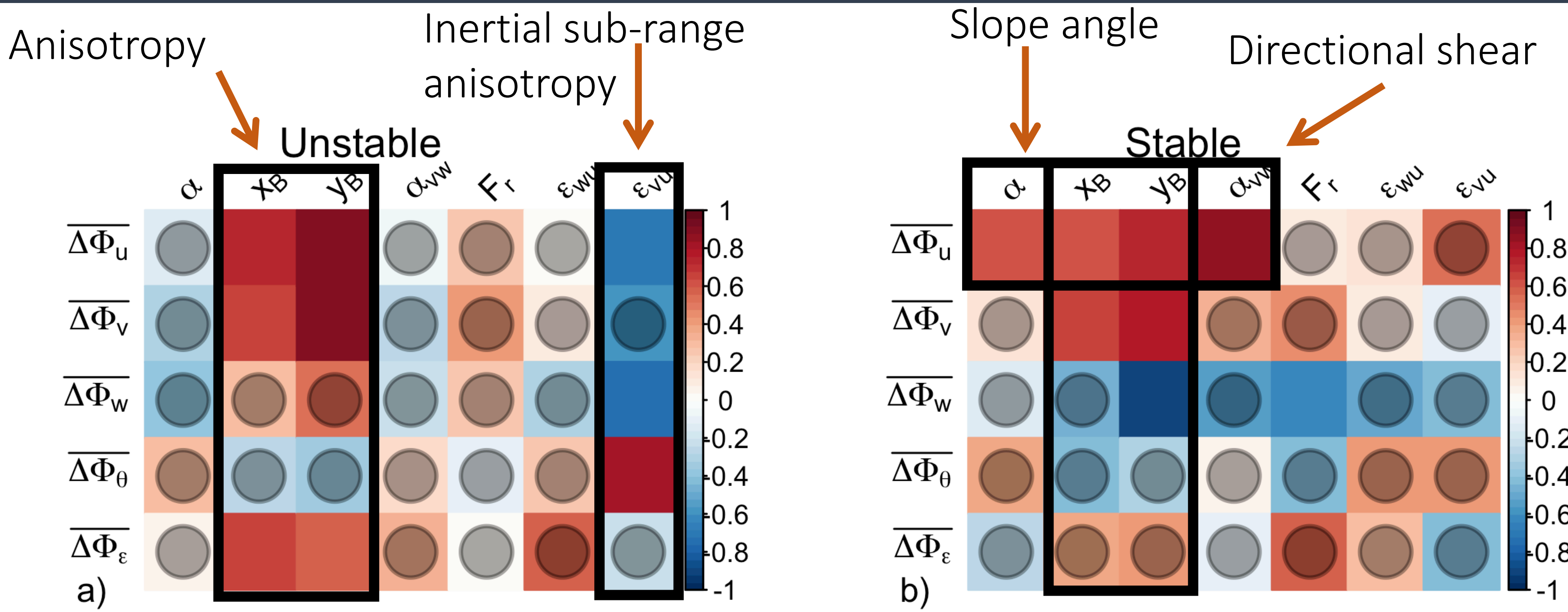




We can define deviations from scaling curve as measure of process we are missing

Stiperski et al. (2019)

Scaling in complex terrain



Multilinear regression between deviation from scaling curve and

- Anisotropy
- Slope angle
- Directional shear

shows these are missing processes

Stiperski et al. (2019)

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Summary: Similarity Theory

- ‘direct approach’ to determine turbulence state
- few (characteristic) scaling variables needed
- ‘recipe’ available
- Scaling regimes
  → Surface Layer (MOST) (→ free convection limit)
  → Mixed Layer
  → Local Scaling Layer (z-less Scaling)
- horizontally homogeneous: established
- complex terrain / heterogeneous surface: only evolving