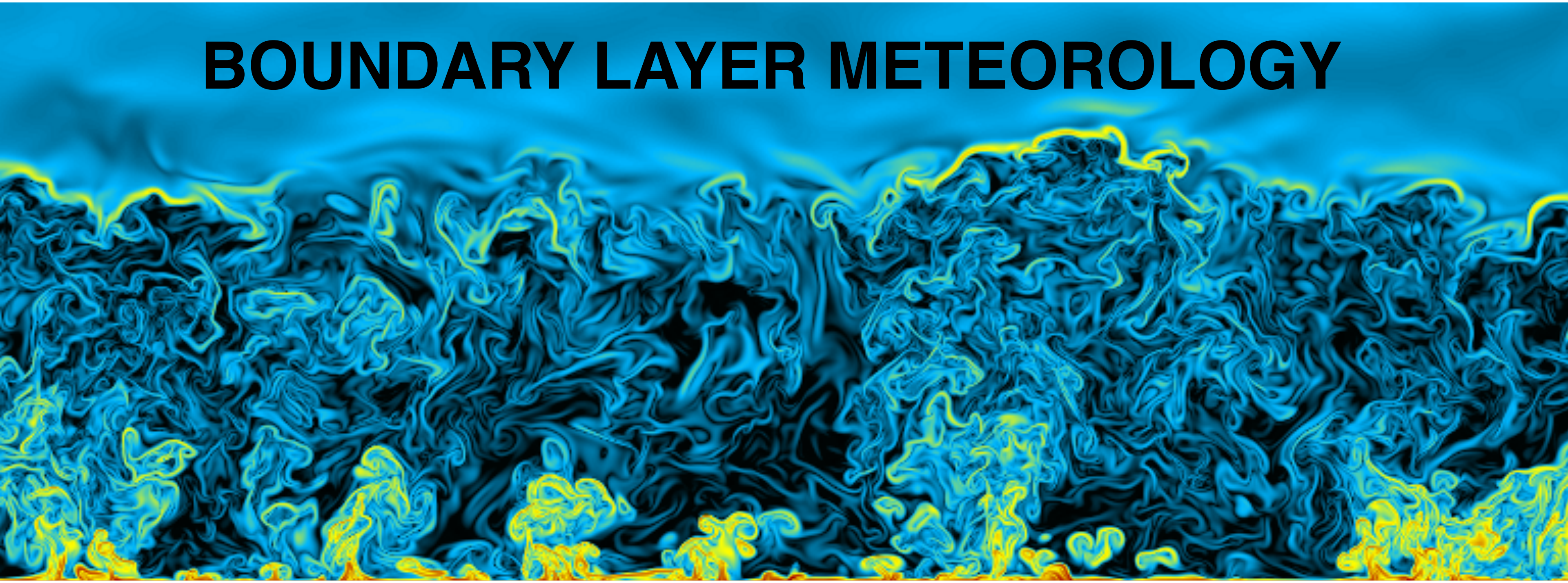
BOUNDARY LAYER METEOROLOGY



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Statistical Treatment of Turbulence

Statistical Treatment of Turbulence:

→Reynolds decomposition and - averaging

→ conservation equations:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\delta_{i3} g + f_c \varepsilon_{ij3} u_j - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial \sigma_{ij}}{\partial x_j}$$

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial}{\partial x_j} \left(\overline{u_i' u_j'} \right) = -\delta_{i3} g + f_c \varepsilon_{ij3} \overline{u_j} - \frac{1}{\overline{\rho}} \frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}}{\partial x_j^2}$$

→ new variables....

2^{nd} Order Moments

→ 2 approaches for treating these new variables

I: Physical approach:
further development of conservation equations

→ simplify (assumptions), solve
→ numerical solutions
→ higher order

II: Similarity theory

$$\overline{w'\theta'},\ \overline{w'q'} \text{ and } \overline{u'w'}, \ldots$$

→ scale analysis
→ *characteristics* of the result?

→ I + II combined (e.g. numerical models, often)

Chapter 4

Similarity Theory

Scaling

Background procedure: *scaling*

Often: different realizations of a process are very different
Often: single realization not very conclusive

→ 'In Bangladesh the prize of 1 kg rice
corresponds to 3 Cts...'

BUT: they are rather similar in terms of dominant,
possibly external variables.

Scaling

Often: single realization not very conclusive

→ 'In Bangladesh the prize of 1 kg rice corresponds to 3 Cts...'

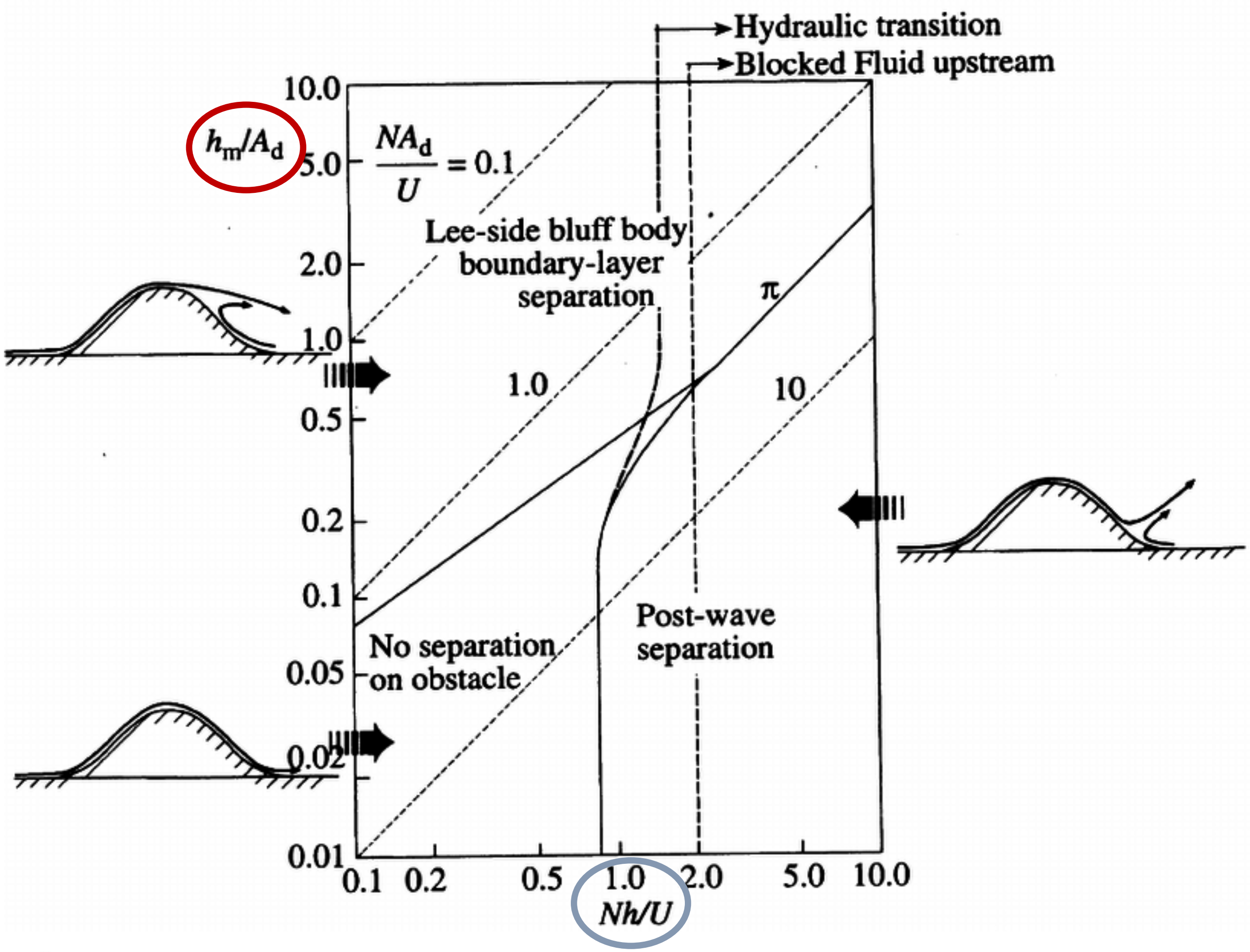
better: $$\frac{\text{price (kg rice)}}{\text{weekly income}} \approx 0.05$$

→ scaling helps comparing similar cases

Scaling example: flow separation

When will we have flow separation from our mountain?

Scaling example: separate or not?



For given aspect ratio

h/A = height/width

It will separate or not depends on:

N – stability

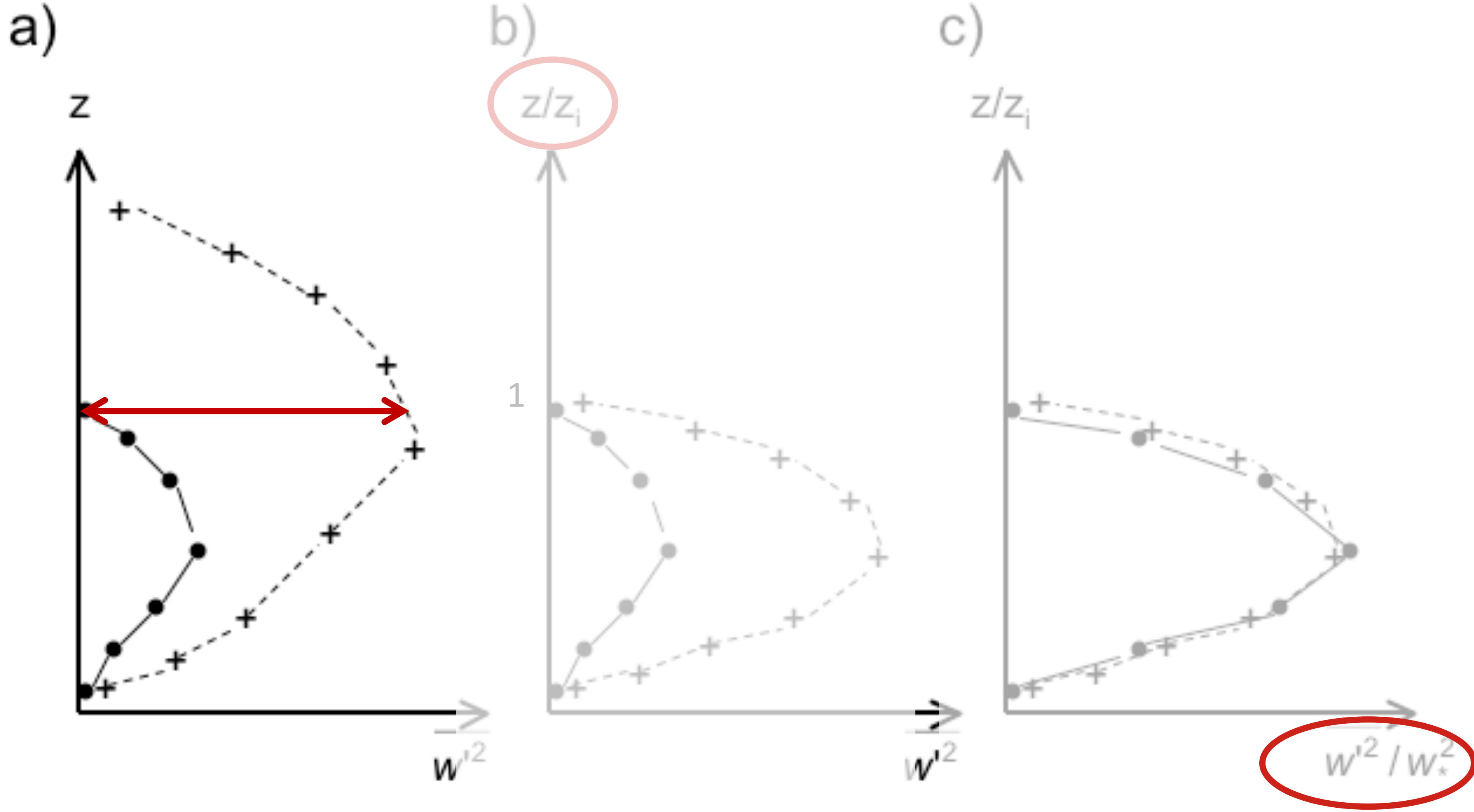
U – wind speed

Baines, 1995

Scaling example: flow separation

When will we have flow separation from our mountain?

Scaling Example: boundary layers



Similarity Theory

→ formal empirical scaling approach for turbulence

Basis of similarity theory: Field experiments

Structure of turbulence over flat terrain
→ shows common, consistent repeatable behavior
→ is determined by a few key processes (variables)

Similarity Theory

Basic hypothesis:
If all the relevant processes are taken into account, any non-dimensional variable can be described as a *universal function* of the maximum number of *independent* non-dimensional combinations of process variables.

Preconditions:
→ equilibrium conditions
(quasi-stationarity, horizontally homogeneous, flat)
→ this is the general case…
→ but is not a necessity (could be relaxed)

Similarity Theory

Approach (recipe):

1. Determine the relevant processes (i.e. the corresponding variables): *one observed variable per process*
2. Determine the maximum number of independent, dimensionless 'π -groups'
   → Buckingham's 'π-Theorem': N π-groups
3. Any mean dimensionless variable in the system, $\overline{a}$, may then be expressed as

$$\frac{\overline{a}}{a_*} = f_a(\pi_{1,}\ \pi_{2,}\ ...\pi_N\ )$$

4. Perform an experiment to determine the shape of f_a

Similarity Theory

Approach (recipe):

→similarity theory does not say anything about the shape of f_a!

→other knowledge (theory) may possibly be used to specify certain conditions for f_a (e.g., limiting values)

Similarity Theory

Approach (recipe):

1. Identify relevant processes:

→ our knowledge (meteorological) of boundary layer
→ under standing physical processes
→ trial and error?
→ which variable is relevant for a process?

Note: only the experiment will tell us, whether we have identified 'all the right' processes under 1) (not too many, none forgotten)

Similarity Theory

Approach (recipe):

1. Determine the relevant processes (i.e. the corresponding variables): *one variable per process*
2. Determine the maximum number of independent, dimensionless 'π -groups'
   → Buckingham's 'π-Theorem': N π-groups
3. Any mean dimensionless variable in the system, $\bar{a}$, may then be expressed as

$$\frac{\bar{a}}{a_*} = f_a(\pi_1, \pi_2, \ldots \pi_N)$$

4. Perform an experiment to determine the shape of f_a

Buckingham π Theory

given: n variables, with r fundamental dimensions

→ [m, kg, s, K, A] °

choose: r *key variables*

whereby: - all fundamental dimensions must be represented in the r variables

- no dimensionless combination of the key variables must be possible

→ determine the 'dimension equations' for the remaining (not 'key') variables:

$$N = n - r$$ equations

Buckingham Pi Theory

→ determine the 'dimension equations' for the remaining (not 'key') variables :

$$N = n - r$$ equations

→ let: $V_1,\ V_2,\ V_3, V_4$ variables

→ be: $V_1 = pressure,\ V_2 = length,$
$V_3 = frequency,\ V_4 = density$ (example)

→ choose: $V_1,\ V_2,\ V_3$ key variables

→ all physical dimensions in V_1, V_2, V_3?

V_1	V_2	V_3	
[kg m^{-1} s^{-2}]	[m]	[s^{-1}]	✓

Buckingham Pi Theory

→ choose: V_1, V_2, V_3 key variables

→ all physical dimensions in V_1, V_2, V_3?

V_1	V_2	V_3	
[kg m^{-1} s^{-2}]	[m]	[s^{-1}]	✓

→ no dimensionless combination possible from V_1, V_2, V_3?

→ 'kg' only in one….. ✓

Buckingham Pi Theory

→ dimension equation:

$$V_4 = (V_1)^a \cdot (V_2)^b \cdot (V_3)^c$$

$$[\mathrm{kg\ m^{-3}}] = [\mathrm{kg\ m^{-1}\ s^{-2}}]^a \cdot [\mathrm{m}]^b \cdot [\mathrm{s^{-1}}]^c$$

→ example:

$$\rho = (\Delta p)^a \cdot (L_*)^b \cdot (f)^c$$

$$[\mathrm{kg\ m^{-3}}] = [\mathrm{kg\ m^{-1}\ s^{-2}}]^a \cdot [\mathrm{m}]^b \cdot [\mathrm{s^{-1}}]^c$$

eq. for kg: $1 = a$

for m: $-3 = -a + b$

for s: $0 = -2a - c$

a=1

b=-2

c=-2

Buckingham Pi Theory

→ determine the exponents
(i.e.: solve the dimension equations)

→ example: a=1, b=-2, c=-2

$$V_4 = \frac{V_1}{V_2^2 V_3^2}$$

→ dimensionally correct

Buckingham Pi Theory

→ for each equation: divide the left hand side by the right hand side

$N = n - r$ π -groups

→ example: $v_4 \cdot \frac{v_2^{\,2} v_3^{\,2}}{v_1} = \pi_1$ → dimensionless

→ $\frac{\bar{a}}{a_*} = f_a(\pi_1, \pi_2, \ldots \pi_N) = f_a(\pi_1)$ → for any mean variable $\bar{a}$ in the system

Buckingham Pi Theory

$$\frac{\bar{a}}{a_*} = f_a(\pi_1, \pi_2, \ldots \pi_N)$$

→ for any mean variable $\bar{a}$ in the system

→ how do I get a_*?

→ use key variables:

→ dimension equation:

$$a_* = (V_1)^e \cdot (V_2)^f \cdot (V_3)^g$$

example:

→ if a_* = scaling **velocity**:

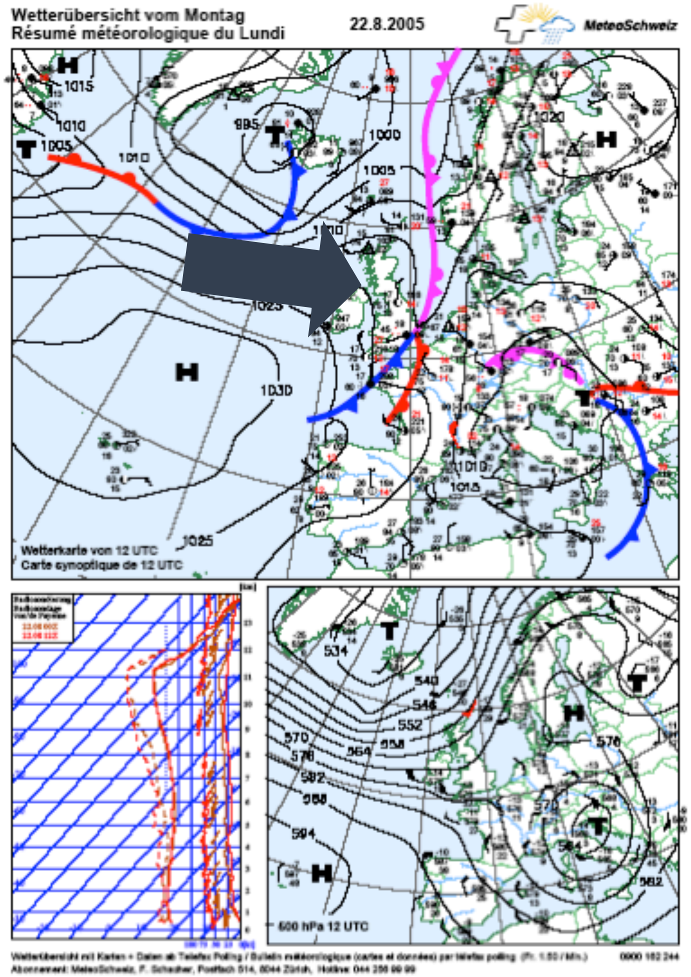
$$[\mathrm{m\,s^{-1}}] = [\mathrm{kg\,m^{-1}\,s^{-2}}]^e \cdot [\mathrm{m}]^f \cdot [\mathrm{s^{-1}}]^g$$

→ e=0

→ f=g=1

$$a_* = V_2 \cdot V_3$$

Example: Geostrophic wind



First order approximation

$$u_G = -\frac{1}{\rho f_c}\frac{\partial p}{\partial y},$$

$$v_G = \frac{1}{\rho f_c}\frac{\partial p}{\partial x}$$

useful: @ synoptic scale

Example: Geostrophic wind

1) Relevant variables:

→ relevant processes: pressure gradient force ($\Delta p/L_*, \rho$)
→ Coriolis acceleration (f)

Δp [kg m^{-1} s^{-2}]

f [s^{-1}]

L_* [m]

ρ [kg m^{-3}]

n=4 variables
r= 3 dimensions

→ 1 dimensionless group

Example: Geostrophic wind

2) Dimensionless groups: N=1

choose: key variables $\Delta p,\ L_*,\ f$

→ no dimensionless group possible (kg in only one variable…)

→ dimension equation: $$\rho = (\Delta p)^a \cdot (L_*)^b \cdot (f)^c$$

$$[\text{kg m}^{-3}] = [\text{kg m}^{-1}\ \text{s}^{-2}]^a \cdot [\text{m}]^b \cdot [\text{s}^{-1}]^c$$

a=1

→ b=-2

c=-2

Example: Geostrophic wind

$$\rho = (\Delta p)^a \cdot (L_*)^b \cdot (f)^c$$

a=1

b=-2

c=-2

$$\longrightarrow \quad \rho = \frac{\Delta p}{L_*^2 f^2}$$

$$\longrightarrow \quad \pi_1 = \frac{\rho L_*^2 f^2}{\Delta p}$$

...divide the left hand side by the right hand side

Example: Geostrophic wind

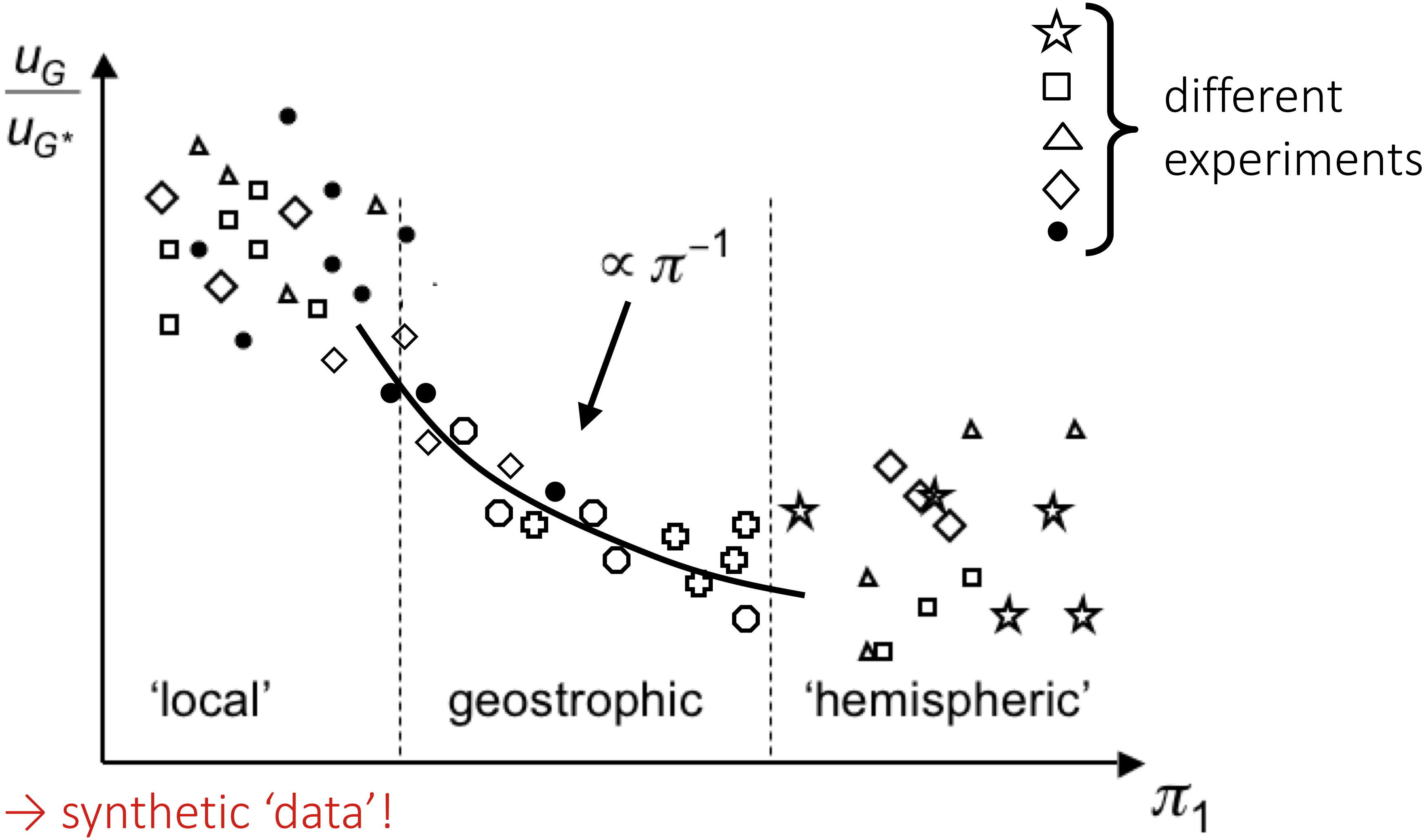
3. Any dimensionless variable in the system, $\overline{a}$, may then be expressed as

$$\frac{\overline{a}}{a_*} = f_a(\pi_1) \qquad (N=1)$$

→ $\overline{u}_G$: geostrophic wind (wanted)
→ $u_{G*} = L_* f$ scaling velocity (produced from the *key variables)*

4. Experiment

Example: Geostrophic wind



→ synthetic 'data'!

Example: Geostrophic wind

→ 'experiment' shows: $f_G(\pi_1) \propto \pi_1^{-1}$

→ $\bar{u}_G \propto u_{G*} \cdot f_G(\pi_1) = fL_* \cdot \frac{\Delta p}{\rho L_*^2 f^2} \cdot c$

$$= c \cdot \frac{1}{\rho f} \frac{\Delta p}{L_*}$$

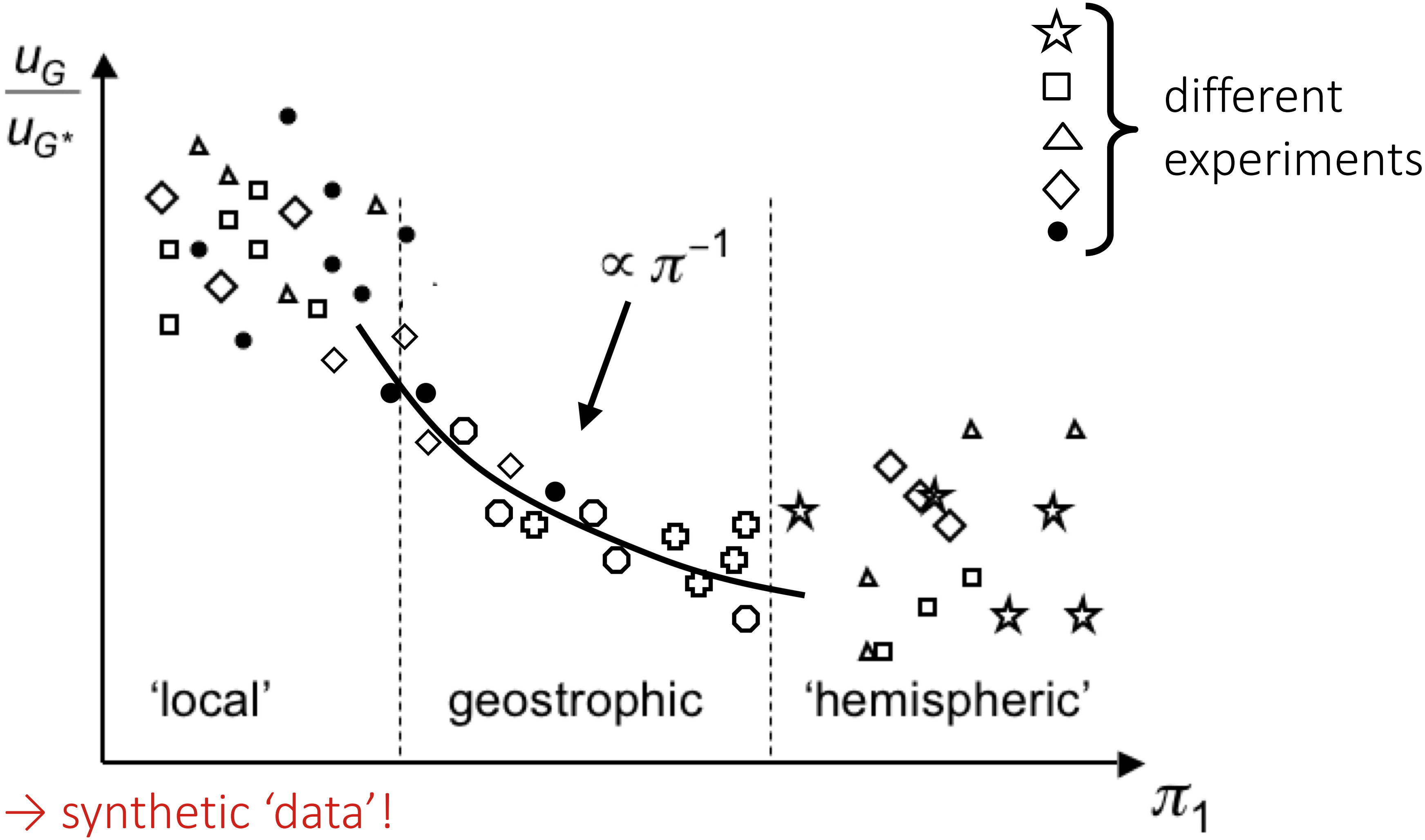
remember:

$$u_G = -\frac{1}{\rho f_c} \frac{\partial p}{\partial y},$$

$$v_G = \frac{1}{\rho f_c} \frac{\partial p}{\partial x}$$

→ 'good' in a certain range of π_1

Example: Geostrophic wind



→ synthetic 'data'!

Example: Period of Pendulum

→ n = 4 dimensional variables:
T (oscillation period), M (mass), L (the length of the string), g (earth gravity)

→ r = 3 fundamental physical units in this equation
time, mass, and length

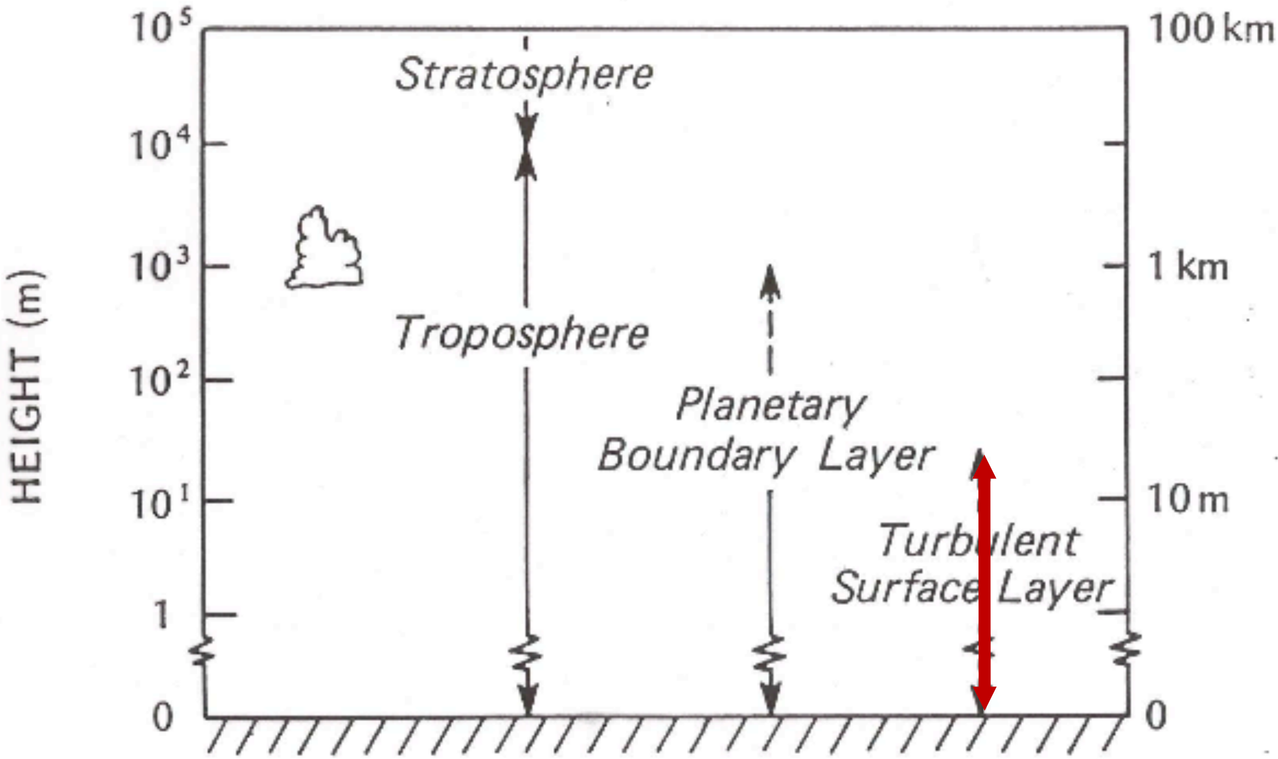
→ we need N = n - r = 4 − 3 = 1 dimensionless quantity

→ dimensionless quantity is: $\Pi = \frac{gT^2}{L}$

and therefore: $\mathrm{T} = const\sqrt{\frac{L}{g}}$

Example: Similarity Theory

Monin-Obukhov Similarity Theory
→ for *surface layer*



assumption SL:
→ fluxes are ~constant with height

→ e.g.: $\overline{u'w'}(z) \approx \overline{u'w'}_o$

Monin-Obukhov Similarity Theory

Relevant processes & variables

→ friction	$\overline{u'w'}_o$	$[m^2 s^{-2}]$
→ heat exchange	$\overline{w'\theta'}_o$	$[mK s^{-1}]$
→ buoyancy	$g/\overline{\theta}$	$[m s^{-2} K^{-1}]$
→ length scale (max size of eddies)	z	$[m]$

→ n=4, r=3 } exactly one dimensionless group (N=1)

Monin-Obukhov Similarity Theory

choice 'key variables':

$\overline{u'w'}_o$ $\overline{w'\theta'}_o$ $g/\overline{\theta}$

$[m^2s^{-2}]$ $[mKs^{-1}]$ $[ms^{-2}K^{-1}]$

→ no dimensionless group possible?

→ would: $[0,0,0] = (\overline{u'w'}_o)^d \cdot (\overline{w'\theta'}_o)^e \cdot (\frac{g}{\overline{\theta}})^f$

eq. for m: $0 = 2d + e + f$ i)

for s: $0 = -2d - e - 2f$ ii)

for K: $0 = e - f$ iii)

Monin-Obukhov Similarity Theory

eq. for m: $0 = 2d + e + f$ i)

for s: $0 = -2d - e - 2f$ ii)

for K: $0 = e - f$ iii)

iii: $e=f$

i: $2d + 2e = 0 \rightarrow d = -e \; (=-f)$

$\rightarrow$ **ii:** $-2d + d + 2d = 0 \leftrightarrow$ d=0 (=e=f)

Monin-Obukhov Similarity Theory

one dimensionless group to determine:

$$z = (\overline{u'w'_o})^a \cdot (\overline{w'\theta'_o})^b \cdot (\frac{g}{\overline{\theta}})^c \qquad \text{(...solve for a, b, c)}$$

$$\longrightarrow \quad z = \frac{(\overline{u'w'_o})^{3/2}}{\frac{g}{\overline{\theta}}\overline{w'\theta'_o}}$$

viz.

$$\pi_1 = \frac{z\frac{g}{\overline{\theta}}\overline{w'\theta'_o}}{(\overline{u'w'_o})^{3/2}}$$

Monin-Obukhov Similarity Theory

for surface layer: $$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

Any mean variable

→ if non-dimensionalised with a_*

→ is a function of π_1 *only*

Example:

→ $\bar{a} = \sigma_w$ (for air pollution modeling...)

→ a_*? → a velocity.....

→ friction velocity (we will see)

$$\frac{\sigma_w}{u_*} = f_w(\pi_1)$$

Monin-Obukhov Similarity Theory

For surface layer: $$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

Monin und Obukhov (1954):

similar approach, but not formally the same
→ what determines turbulence near the surface?
→ friction and heat exchange

friction → momentum flux
heat exchange → sensible heat flux

Monin-Obukhov Similarity Theory

For surface layer: $$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

Monin und Obukhov (1954):

surface layer: turbulent flux at the surface ($\overline{u'w'}_o$, $\overline{w'\theta'}_o$) is characteristic for the SL

surface layer = *'constant flux layer'*
→ turbulent fluxes *do not significantly change* over lowest 10% of boundary layer
→ surface fluxes influence (determine) turbulence in the entire SL

Monin-Obukhov Similarity Theory

For surface layer: $$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

Monin und Obukhov (1954):

Def: $$u_* \equiv (-\overline{u'w'_o})^{1/2}$$ characteristic velocity

$$\theta_* \equiv -\overline{w'\theta'_o} / u_*$$ characteristic temperature

$$L \equiv \frac{1}{k}\frac{u_*^2}{\theta_*}\left(\frac{g}{\overline{\theta}}\right)^{-1} = -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'_o}}\left(\frac{g}{\overline{\theta}}\right)^{-1}$$ characteristic length

Monin-Obukhov Similarity Theory

For surface layer: $$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

Monin und Obukhov (1954):

Def: $u_* =: (-\overline{u'w'}_o)^{1/2}$ characteristic velocity

note: friction velocity was defined earlier:

$$u_* \equiv \left(\overline{u'w'}_o^2 + \overline{v'w'}_o^2 \right)^{1/4} \qquad (*)$$

→ in streamline–coordinates: $\overline{v'w'} = 0$

→ (*) is proper definition (should also generally be in L)

Monin-Obukhov Similarity Theory

For surface layer:

$$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

$$\frac{\bar{a}}{a_*} = f_a\left(\frac{z}{L}\right)$$

Monin und Obukhov (1954):

$$L = \frac{1}{k}\frac{u_*^2}{\theta_*}\left(\frac{g}{\bar{\bar{\theta}}}\right)^{-1} = -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'}_o}\left(\frac{g}{\bar{\bar{\theta}}}\right)^{-1}$$

compare

our analysis

$$\pi_1 = \frac{z\frac{g}{\bar{\bar{\theta}}}\overline{w'\theta'_o}}{\left(\overline{u'w'_o}\right)^{3/2}} \approx \frac{z}{L}$$

→ difference:

von Kàrmàn constant **k**

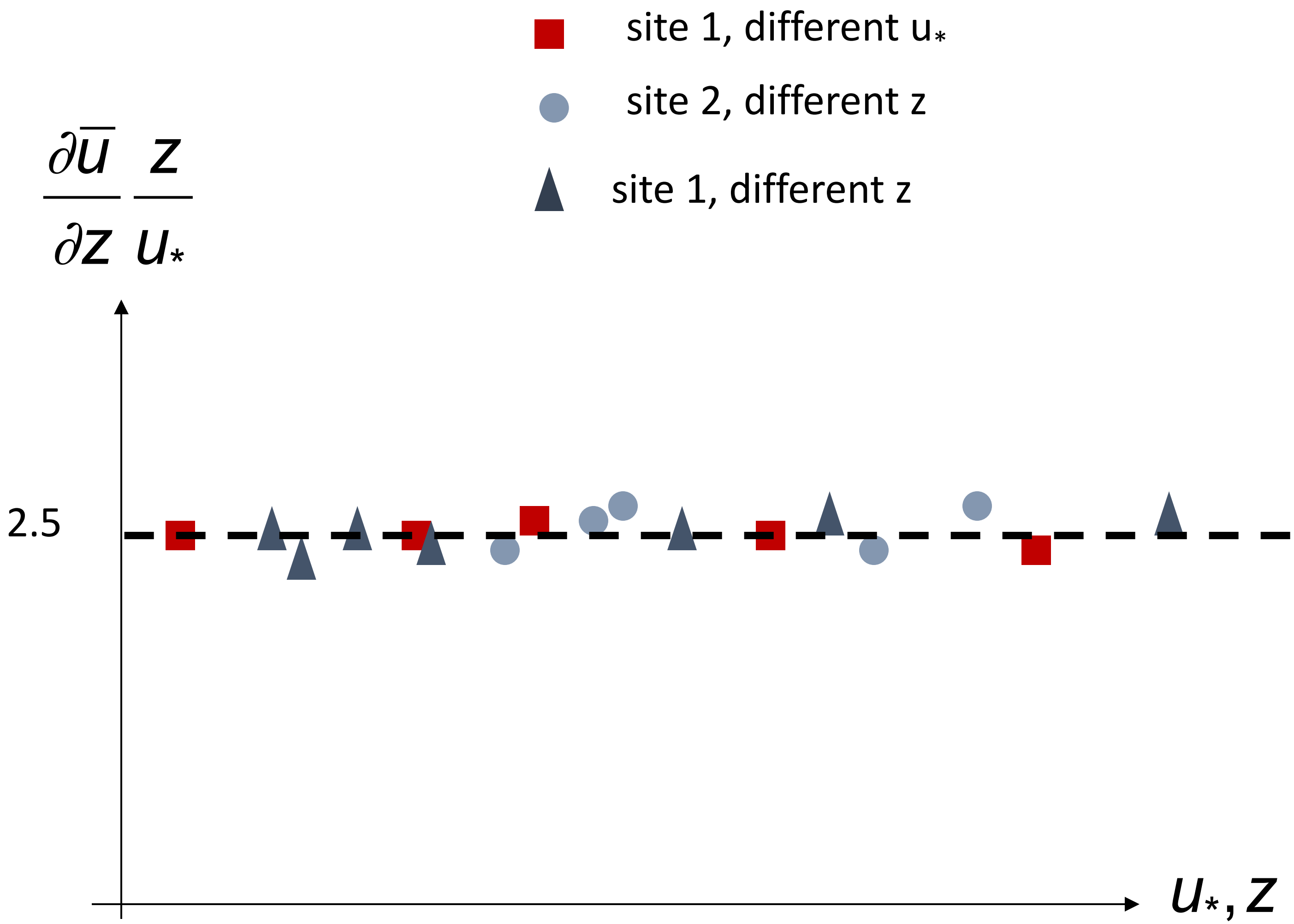
Van Kàrmàn constant

Early approach (before MOST):

$$\frac{\partial \overline{u}}{\partial z}\frac{z}{u_*} = const.$$

→ experiments: 'const' =1/0.4=2.5

Van Kàrmàn constant



Van Kàrmàn constant

Early approach (before MOST):

$$\frac{\partial \overline{u}}{\partial z}\frac{z}{u_*} = const.$$

→ experiments: const =1/0.4

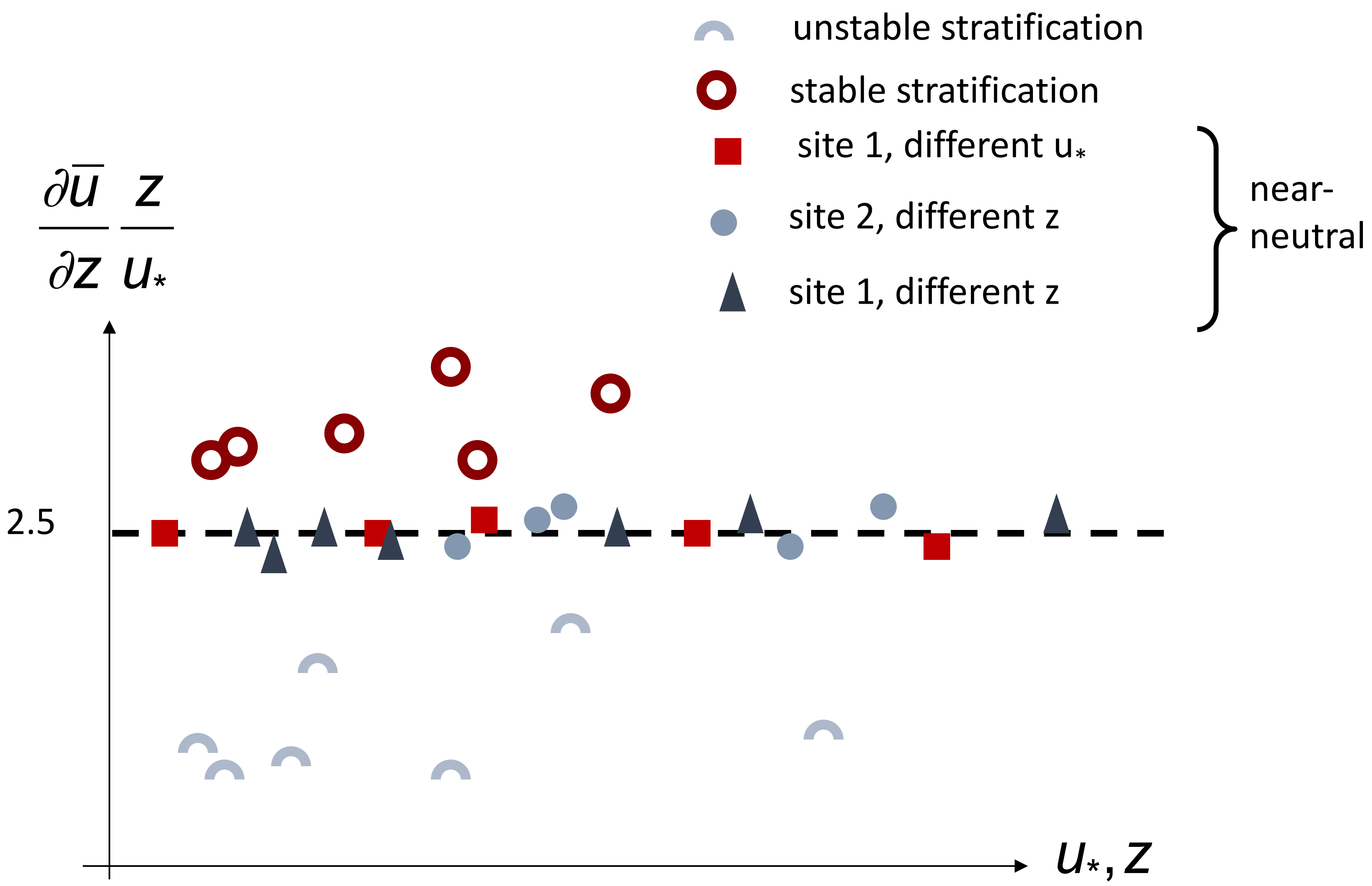
$$\frac{\partial \overline{u}}{\partial z}\frac{z}{u_*}0.4 = 1$$

k

$$\frac{\partial \overline{u}}{\partial z}\frac{kz}{u_*} = 1$$

However: → only constant for near-neutral flows
→ stable flows: 1/k larger
→ unstable flows: 1/k smaller

Van Kàrmàn constant



Van Kàrmàn constant

early approach (before MOST):

$$\frac{\partial \bar{u}}{\partial z}\frac{z}{u_*} = const.$$

→ experiments: const =1/0.4

$$\frac{\partial \bar{u}}{\partial z}\frac{z}{u_*}0.4 = 1$$

(0.4 ← k)

However: → only constant for near-neutral flows
→ stable flows: 1/k larger
→ unstable flows: 1/k smaller

Any new theory (MOST):
→ retain success of 'old theory' (neutral conditions)
→ better & more general where old theory fails

→ keep k in MOST

Monin-Obukhov Similarity Theory

→ for *Surface Layer*
→ dependence on z/L
→ Obukhov Length L:

$$\frac{\overline{a}}{a_*} = f_a(\pi_1)$$

$$\frac{z}{L} = -\frac{zk\frac{g}{\overline{\theta}}\overline{w'\theta'_o}}{(\overline{u'w'_o})^{3/2}}$$

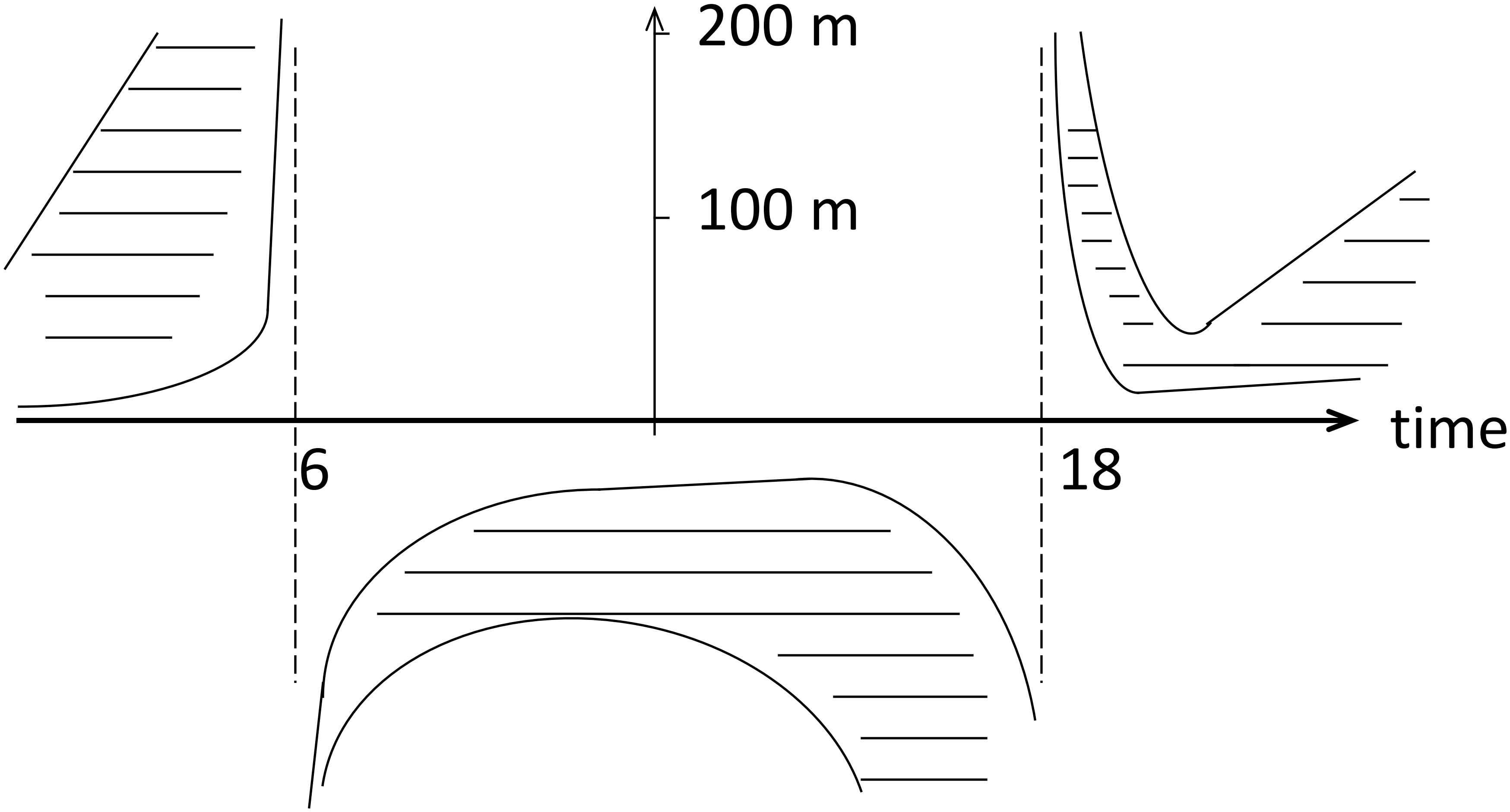
measure for stability of stratification

$\overline{w'\theta'_o} = 0 \quad \Rightarrow \quad L = \infty$ z/L=0: neutral

$\overline{w'\theta'_o} > 0 \quad \Rightarrow \quad L < 0$ z/L<0: unstable

$\overline{w'\theta'_o} < 0 \quad \Rightarrow \quad L > 0$ z/L>0: stable

Obukhov Length L



Surface Layer Scaling (MOST)

$$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

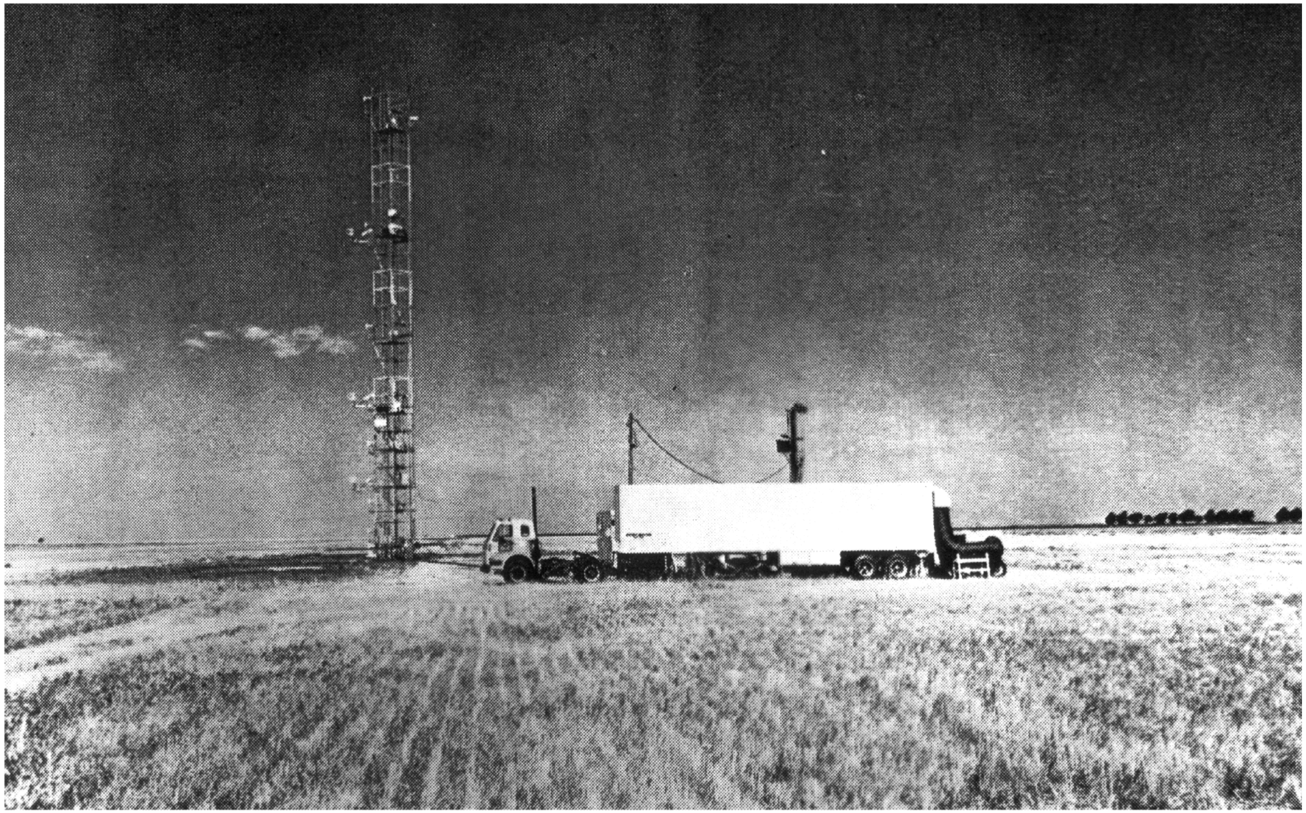
→ make an experiment to determine f_a

example
wind speed:

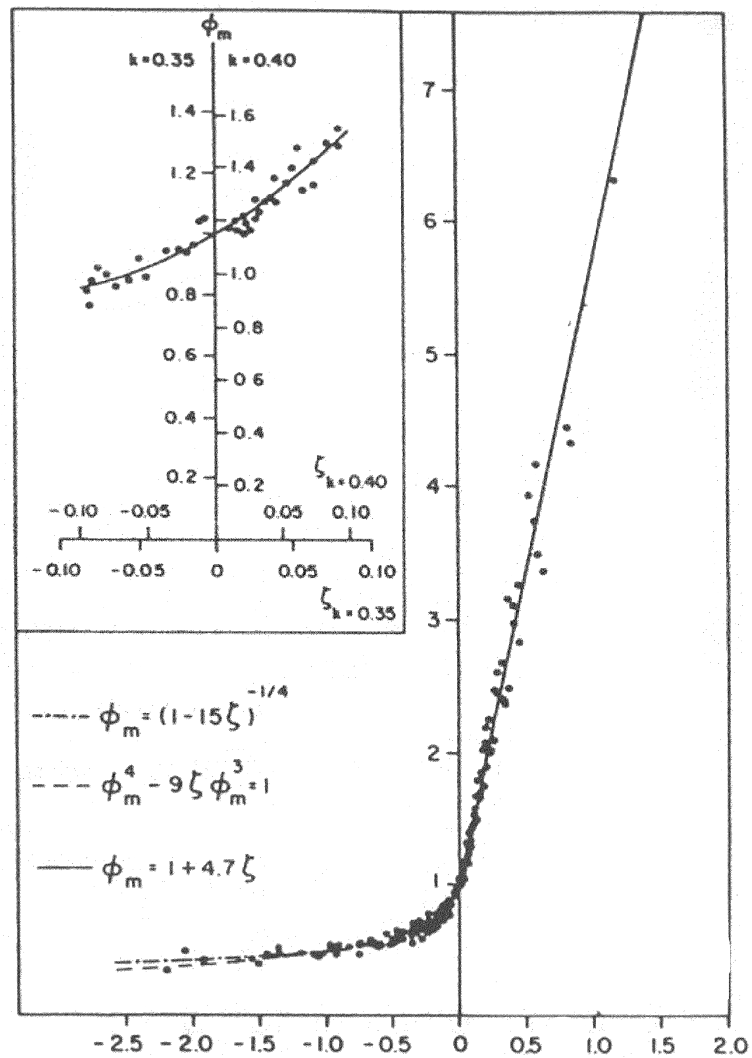
$$\underbrace{\frac{\partial \bar{u}}{\partial z}}_{\bar{a}} \underbrace{\frac{kz}{u_*}}_{a_*} = \phi_m(z / L)$$

Surface Layer Scaling (MOST)

Designing your experiment



Kansas Experiment 1969



→ experiment says, whether scaling is ok ....

→ here: apparently ok

Businger et al (1971)

Surface Layer Scaling (MOST)

$$\frac{\overline{a}}{a_*} = f_a\left(\frac{z}{L}\right)$$

wind:

$$\frac{\partial \overline{u}}{\partial z}\frac{kz}{u_*} = \phi_m(z/L)$$

T-variance:

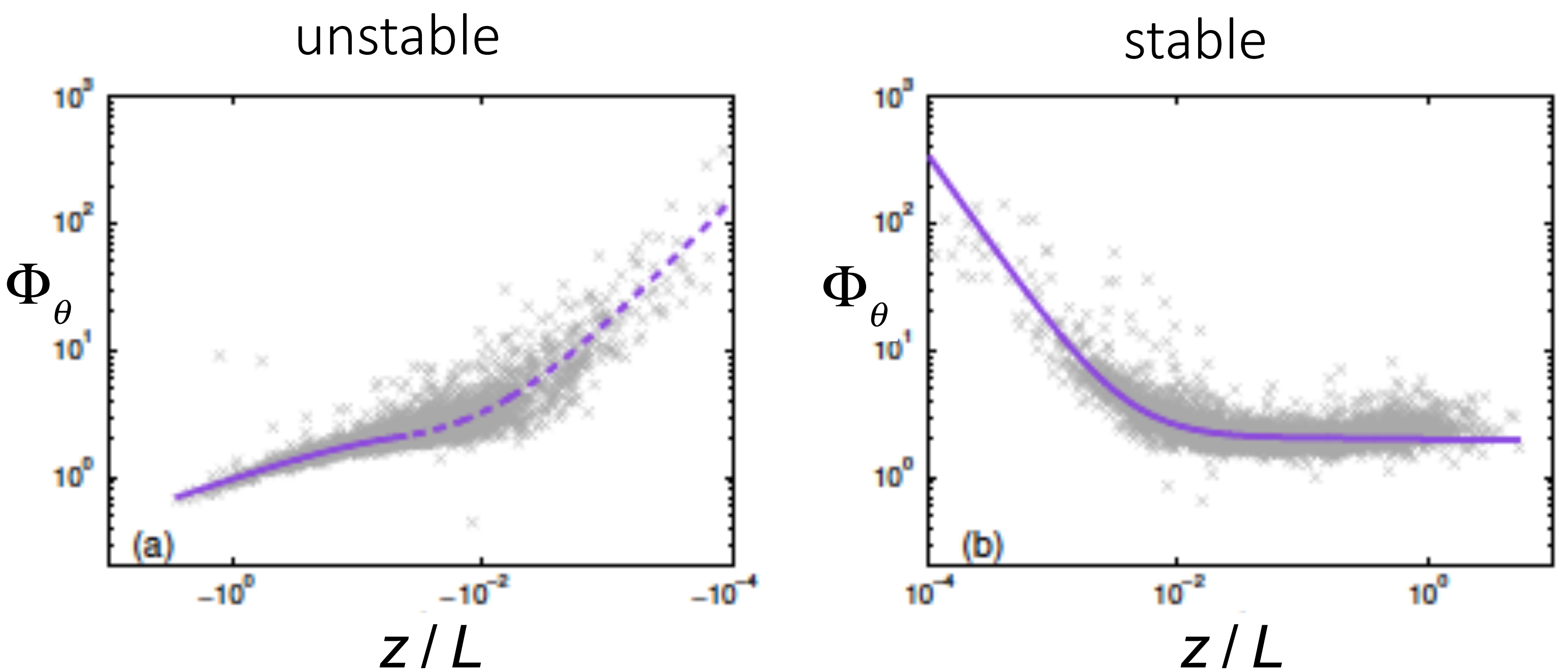
$$\frac{\sigma_\theta}{\theta_*} = \phi_\theta\left(\frac{z}{L}\right)$$

$$\theta_* = -\overline{w'\theta'}_o / u_*$$

characteristic temperature

Surface Layer Scaling (MOST)

Non-dimensional temperature variance



$$\Phi_\theta = \frac{\sigma_\theta}{|\theta_*|}$$

- data from Cabauw observatory
- near neutral: $\overline{w'\theta'} \rightarrow 0$

Sfyri et al. (2018)

Surface Layer Scaling (MOST)

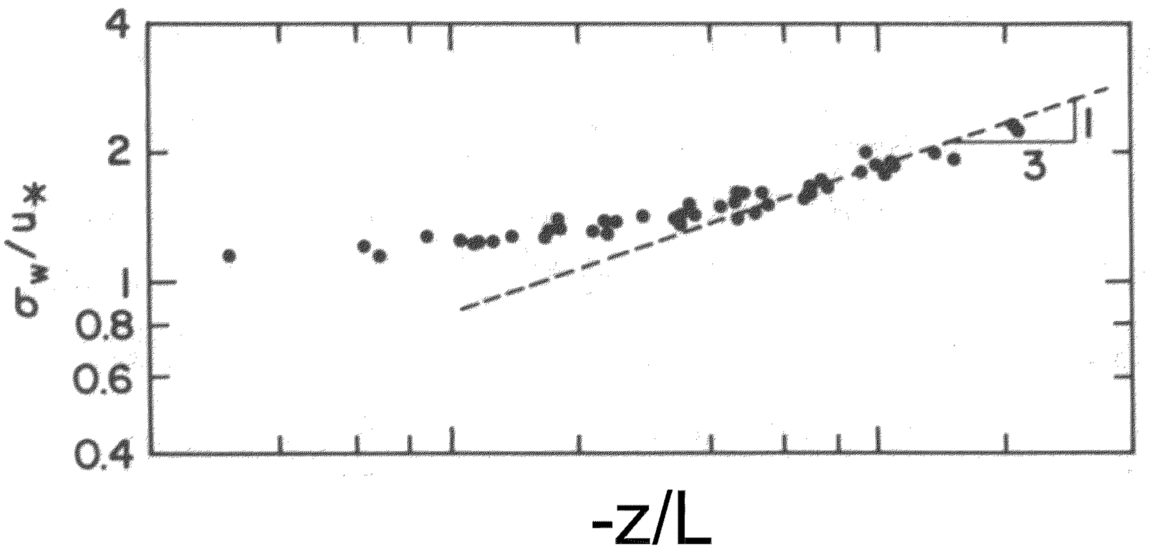
$$\frac{\overline{a}}{a_*} = f_a\left(\frac{z}{L}\right)$$

wind: $$\frac{\partial \overline{u}}{\partial z}\frac{kz}{u_*} = \phi_m(z/L)$$

T-variance: $$\frac{\sigma_\theta}{\theta_*} = \phi_\theta\left(\frac{z}{L}\right)$$

velocity variance $$\frac{\sigma_w}{u_*} = \phi_w\left(\frac{z}{L}\right)$$

Surface Layer Scaling (MOST)



- Kansas
- unstable
- ‚free convection limit': -1/3 power

$$\frac{\sigma_w}{u_*} = 1.3\left(1-3\frac{z}{L}\right)^{1/3}$$

Surface Layer Scaling (MOST)

$$\frac{\overline{a}}{a_*} = f_a\left(\frac{z}{L}\right)$$

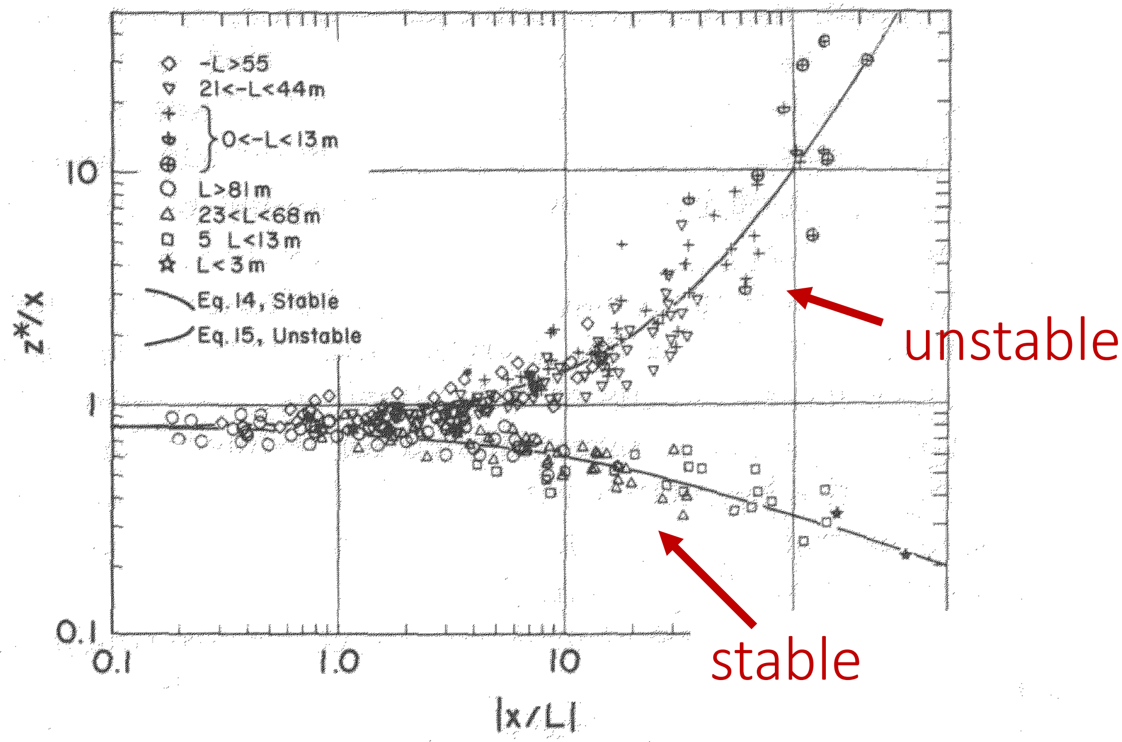
wind: $$\frac{\partial \overline{u}}{\partial z}\frac{kz}{u_*} = \phi_m(z/L)$$

T-variance: $$\frac{\sigma_\theta}{\theta_*} = \phi_\theta\left(\frac{z}{L}\right)$$

velocity variance: $$\frac{\sigma_w}{u_*} = \phi_w\left(\frac{z}{L}\right)$$

Tracer concentration: $$\frac{\chi_*}{x} = f_\chi\left(\left|\frac{x}{L}\right|\right)$$

Surface Layer Scaling (MOST)



- Prairie Grass, tracer experiments
- $z_* = Q/(u_* \int \chi dy)$caled concentration
- transformation of z/L dependence into x/L through u

Surface Layer Scaling (MOST)

$$\frac{\bar{a}}{a_*} = f_a(\pi_1)$$

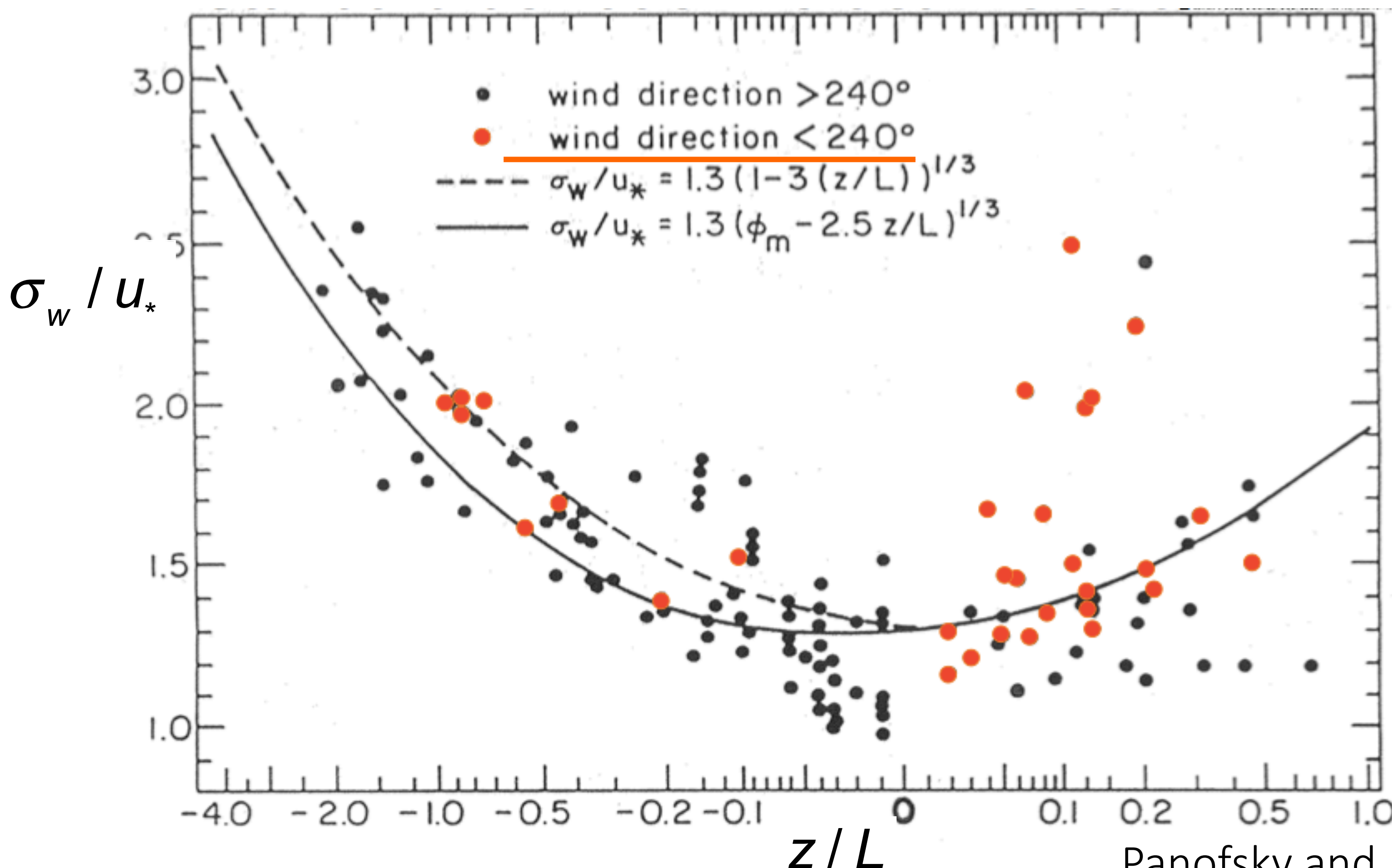
Note: only the experiment will tell us, whether
we have identified 'all the right' processes
under 1) (not too many, none forgotten)

→ example: vertical velocity variance
→ in ‚Kansas': yes
→ in hilly terrain; no....

$$\frac{\sigma_w}{u_*} = \phi_w\left(\frac{z}{L}\right)$$

Surface Layer Scaling (MOST)

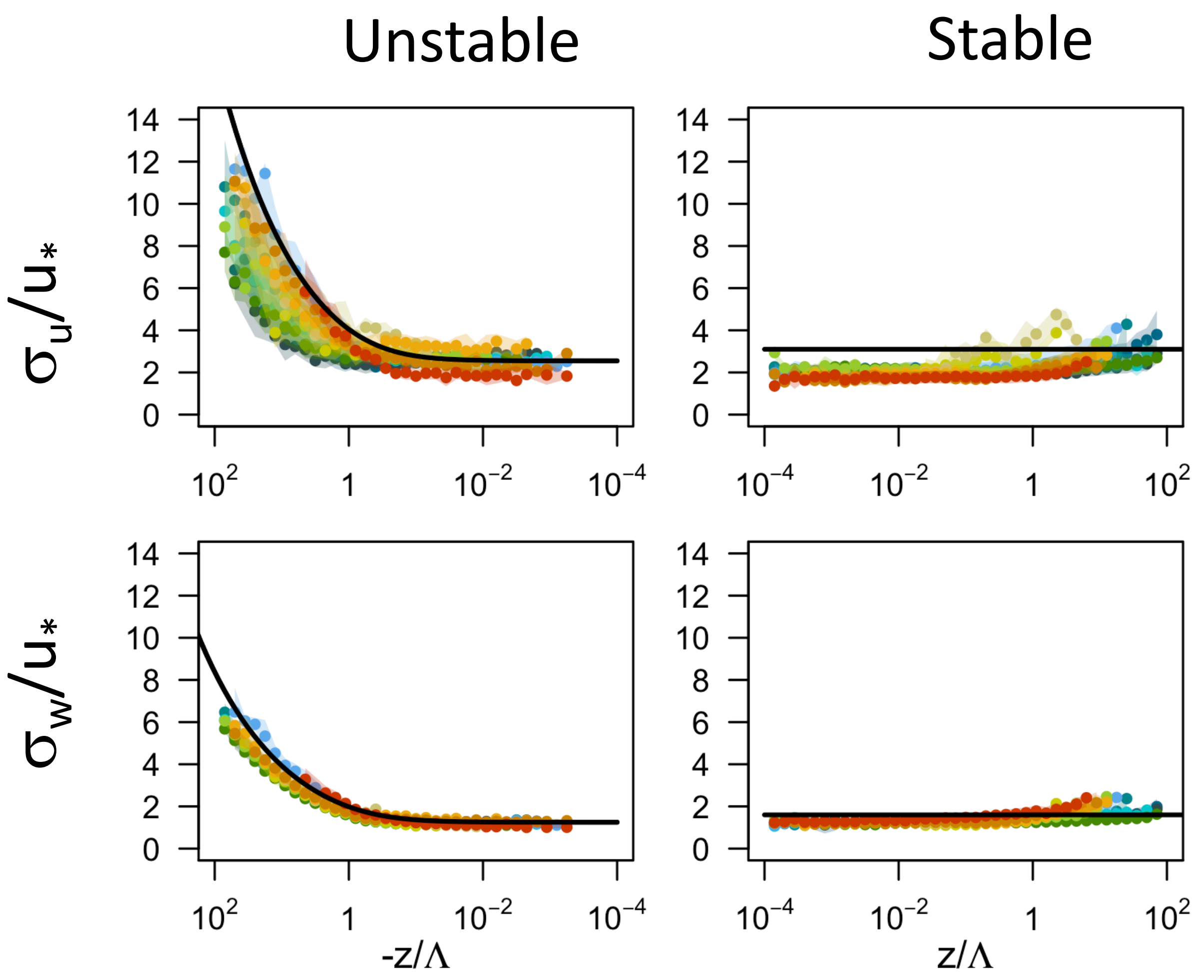
Standard deviation vertical velocity



Panofsky and Dutton (1984)

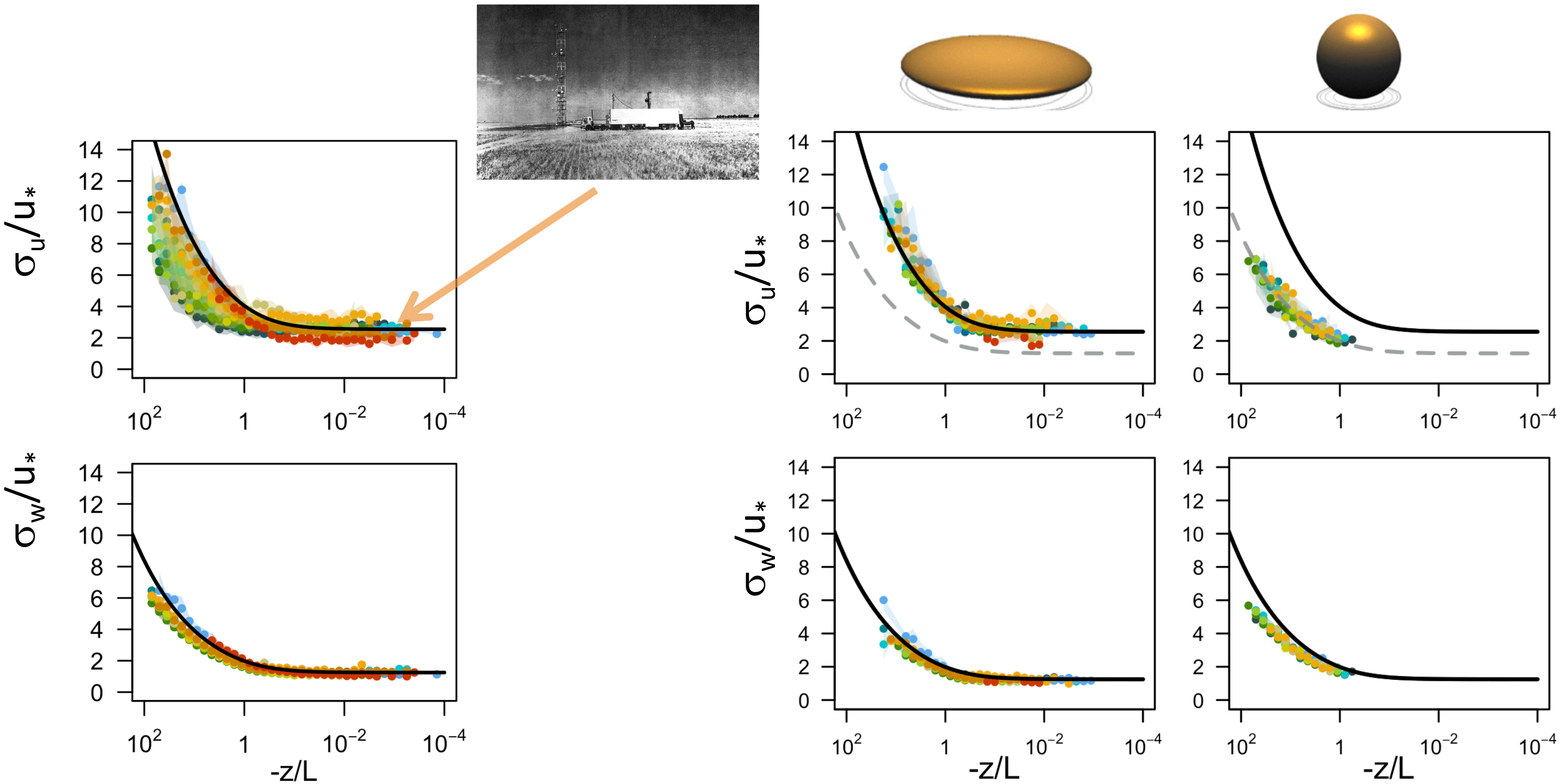
Surface Layer Scaling (MOST)

Standard deviation horizontal velocity



Stiperski et al. (2019)

Standard deviation horizontal velocity

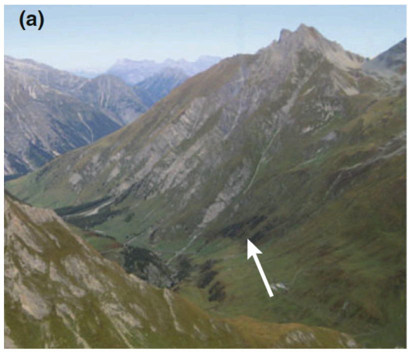


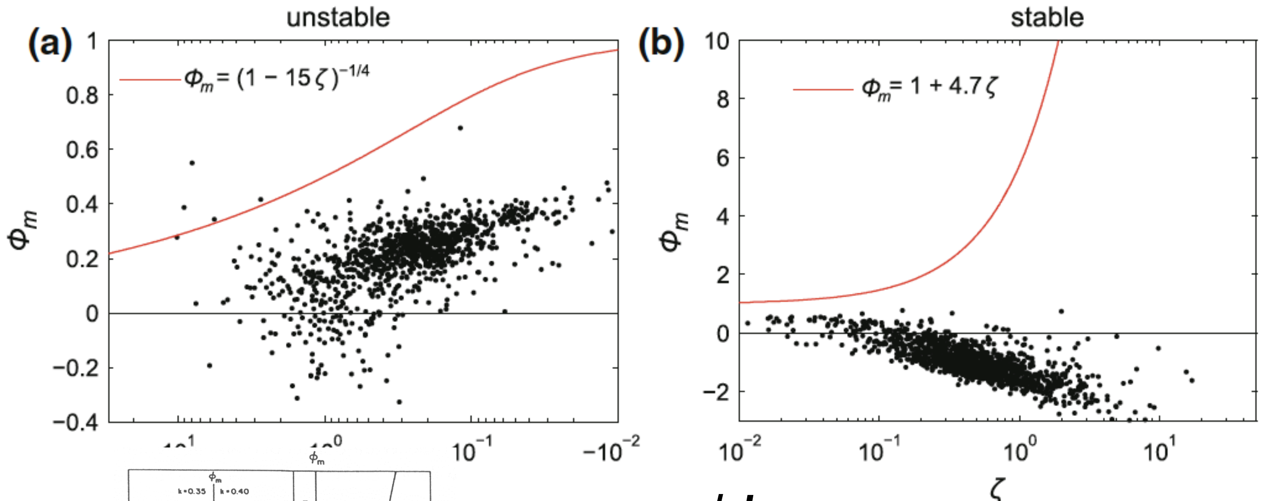
Stiperski et al. (2019)

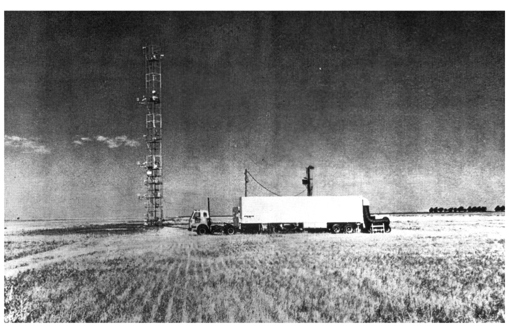
Surface Layer Scaling (MOST)

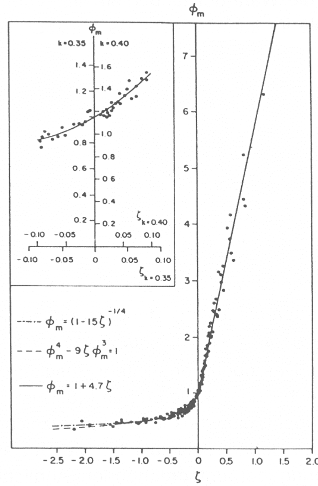
Wind profile

$$\phi_m(z/L) = \frac{\partial \bar{u}}{\partial z}\frac{kz}{u_*}$$









z/L

Nadeau et al (2013)

Surface Layer Scaling (MOST)

Use MOST (for the Surface Layer)

get wind profile in the SL:
→ based on MOST prediction for non-dim gradient

wind profile: $$\frac{\partial \overline{u}}{\partial z}\frac{kz}{u_*} = \phi_m(z/L)$$

Surface Layer Scaling (MOST)

wind: $$\frac{\partial \bar{u}}{\partial z}\frac{kz}{u_*} = \phi_m(z/L)$$

let: $$\phi_m(z/L=0)=1$$ (choice of k! → k=0.4)
→ neutral!

$$\longrightarrow \quad d\bar{u} = \frac{u_*}{k}\frac{dz}{z} \qquad \longrightarrow \quad \int_{u_1}^{u_2} d\bar{u} = \frac{u_*}{k}\int_{z_1}^{z_2}\frac{dz}{z}$$

$$\longrightarrow \quad \bar{u}(z_2)-\bar{u}(z_1) = \int_{u_1}^{u_2} d\bar{u} = \frac{u_*}{k}\int_{z_1}^{z_2}\frac{dz}{z} = \frac{u_*}{k}\ln\left(\frac{z_2}{z_1}\right)$$

Def: $$\bar{u}(z=z_o)\equiv 0$$ z_o: roughness length

$$\bar{u}(z) = \frac{u_*}{k}\ln\left(\frac{z}{z_o}\right)$$ → log profile, neutral

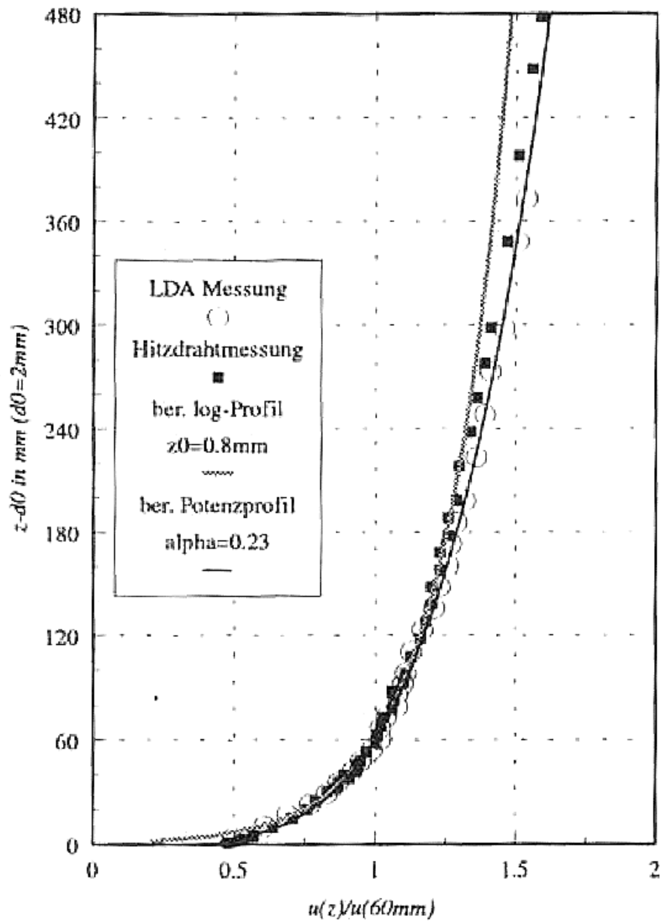
Surface Layer Scaling (MOST)

$$\overline{u}(z) = \frac{u_*}{k} \ln\left(\frac{z}{z_o}\right)$$

→ log profile, neutral

wind tunnel observation

→ ideally neutral

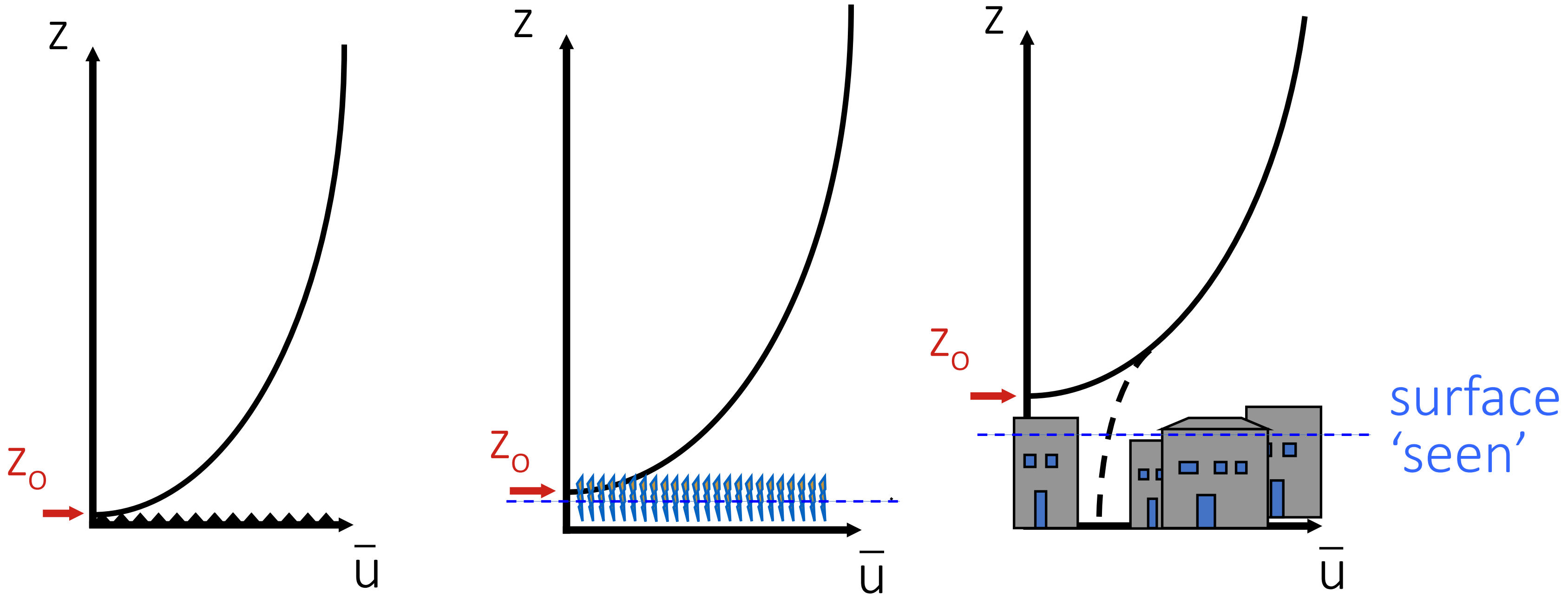


Surface Layer Scaling (MOST)

Wind profile Surface Layer

$$\overline{u}(z) = \frac{u_*}{k} \ln\left(\frac{z}{z_o}\right)$$

→ $u(z_o)=0$



→ z_o dependent on size of roughness elements, $z_o \approx 0.1h$

→ log-profile is a property of Surface Layer

Surface Layer Scaling (MOST)

Wind: $$\frac{\partial \bar{u}}{\partial z}\frac{kz}{u_*} = \phi_m(z/L)$$

non-neutral: $$\frac{k}{u_*}\int_0^{\bar{u}} d\bar{u}' = \int_{z_o}^{z} \frac{\phi_m(z'/L)}{z'}dz'$$

$$= \int_{z_o}^{z} \frac{dz'}{z'} - \int_{z_o}^{z} \frac{1-\phi_M(z'/L)}{z'}dz'$$

$$= \ln\left(\frac{z}{z_o}\right) - \int_{z_o}^{z} \frac{1-\phi_M(z'/L)}{z'}dz'$$

Surface Layer Scaling (MOST)

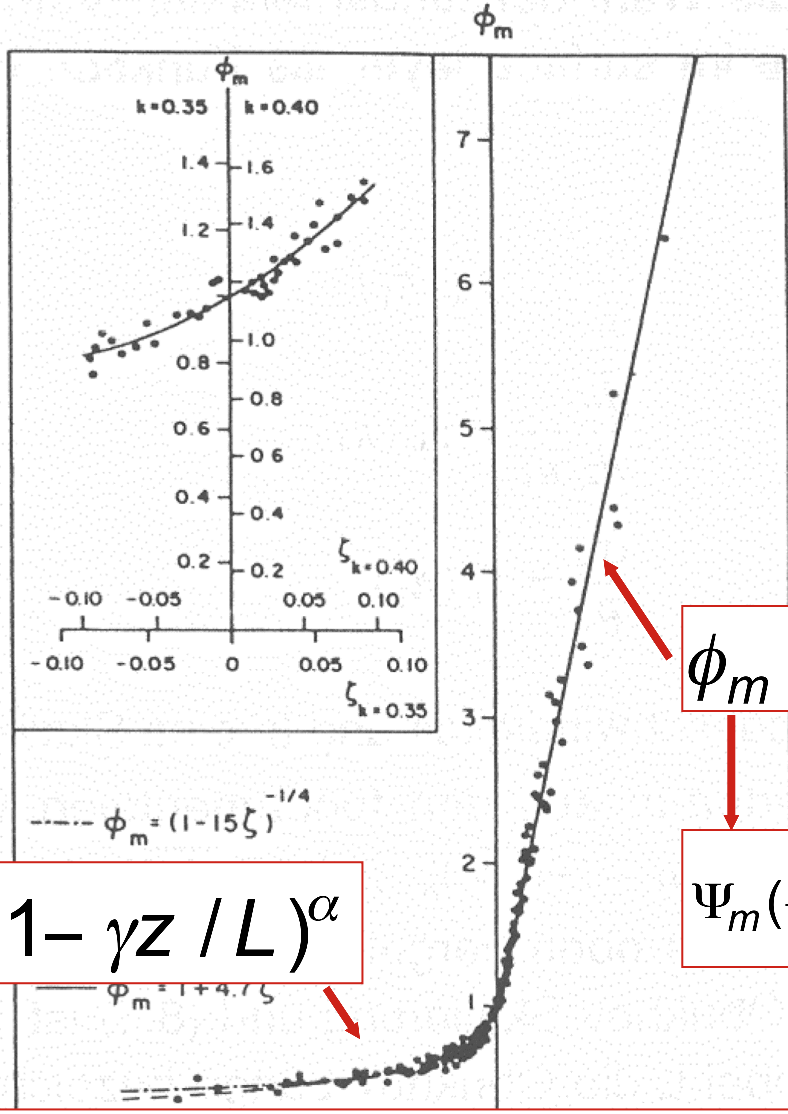
$$\frac{k}{u_*}\int_0^{\bar{u}} d\bar{u}' = \ln\left(\frac{z}{z_o}\right) - \int_{z_o}^{z} \frac{1-\phi_M\left(z'/L\right)}{z'}dz'$$

$$\underbrace{\int_{z_o}^{z} \frac{1-\phi_M\left(z'/L\right)}{z'}dz'}_{\equiv \Psi_m\left(z/L\right)}$$

$$\longrightarrow \bar{u}\left(z\right) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) - \Psi_m\left(z/L\right)\right]$$

'correction' for stability

Z/L



$$\phi_m = 1 + \beta z / L$$

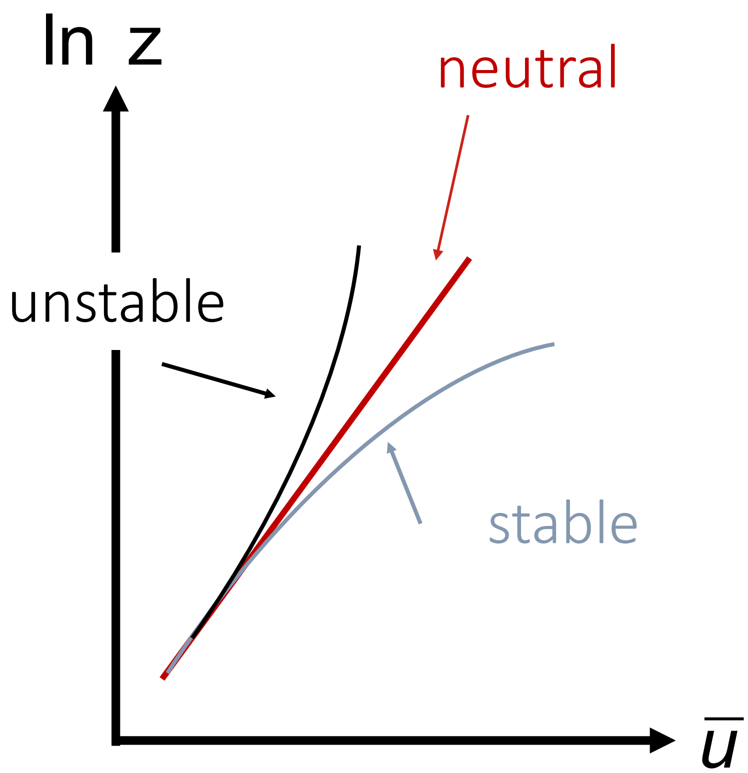
$$\Psi_m(\frac{z}{L}) = -\beta \frac{z}{L}$$

$$\phi_m = (1 - \gamma z / L)^{\alpha}$$

$$\Psi_m(\frac{z}{L}) = -2\ln\left[\frac{1+x}{2}\right] - \ln\left[\frac{1+x^2}{2}\right] + 2\tan^{-1}(x) - \frac{\pi}{2} \quad (x = (1 - \gamma z / L)^{\alpha})$$

Surface Layer Scaling (MOST)

wind: $$\bar{u}(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) - \Psi_m(z/L)\right]$$



Surface Layer Scaling (MOST)

Potential temperature:

$$\frac{\partial \overline{\theta}}{\partial z}\frac{kz}{\theta_*} = \phi_H(z/L)$$

non-neutral:

$$\frac{k}{\theta_*}\int_{\overline{\theta}(z_{oH})}^{\overline{\theta}(z)} d\overline{\theta}' = \int_{z_{oH}}^{z} \frac{\phi_H(z'/L)}{z'} dz'$$

$$= \int_{z_{oH}}^{z} \frac{dz'}{z'} - \int_{z_{oH}}^{z} \frac{1-\phi_H(z'/L)}{z'} dz'$$

$$= \ln\left(\frac{z}{z_{oH}}\right) - \int_{z_{oH}}^{z} \frac{1-\phi_H(z'/L)}{z'} dz'$$

Surface Layer Scaling (MOST)

$$\frac{k}{\theta_*}\int_{\bar{\theta}(z_{oH})}^{\bar{\theta}(z)} d\bar{\theta}' = \ln\left(\frac{z}{z_{oH}}\right) - \underbrace{\int_{z_{oH}}^{z} \frac{1-\phi_H\left(z'/L\right)}{z'}dz'}_{\equiv \Psi_H\left(z/L\right)}$$

$$\longrightarrow \quad \bar{\theta}\left(z\right) - \bar{\theta}\left(z_{oH}\right) = \frac{\theta_*}{k}\left[\ln\left(\frac{z}{z_{oH}}\right) - \Psi_H\left(z/L\right)\right]$$

→ z_{oH}: ‚reference height‘, ‚roughness length for temperature‘
→ not the same as z_o
→ some ‚10 times smaller than z_o‘

Surface Layer Scaling (MOST)

$$\rightarrow \quad \bar{\theta}(z) - \bar{\theta}(z_{oH}) = \frac{\theta_*}{k}\left[\ln\left(\frac{z}{z_{oH}}\right) - \Psi_H(z/L)\right]$$

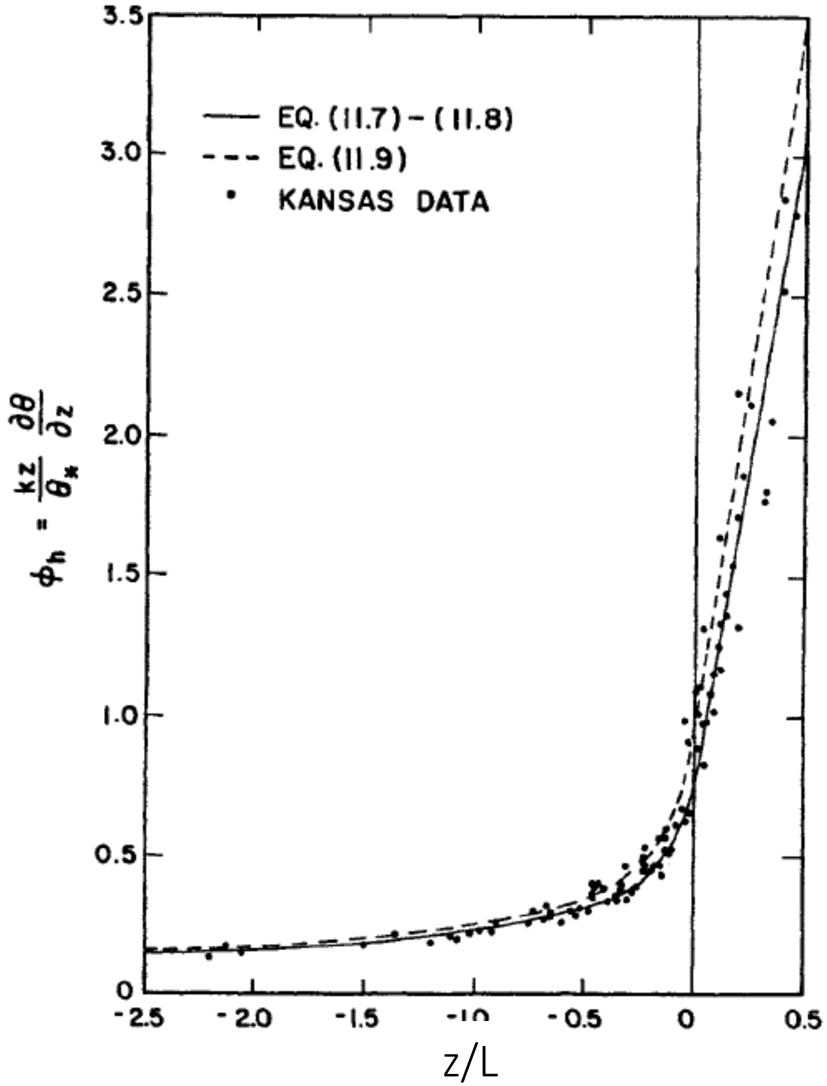
general form:

$$\frac{z}{L} < 0 \qquad \phi_H(z/L) = (1-15\frac{z}{L})^{-1/2} \qquad \Psi_H(z/L) = 2\ln[\frac{1+x^2}{2}]$$

$$x = (1-15\frac{z}{L})^{1/4}$$

$$\frac{z}{L} > 0 \qquad \phi_H(z/L) = (1+6\frac{z}{L}) \qquad \Psi_H(z/L) = -6z/L$$

Surface Layer Scaling (MOST)



Businger et al (1971)

MOST: Free convection limit

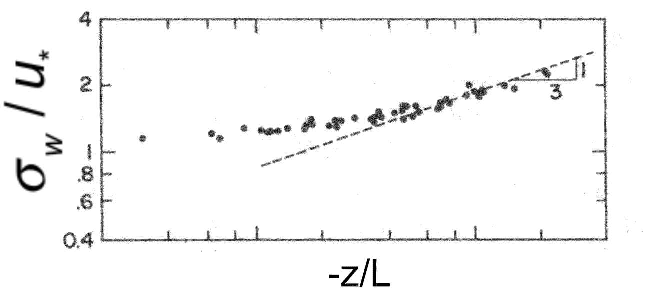
Free convection limit:
→ at large -z/L, friction becomes unimportant
→ limiting behaviour: independent of u_*
→ can use this to deduce 'limit of MOST similarity functions'

example $\sigma_w / u_* = \phi_w(z/L)$

re-write: $\sigma_w = u_* \phi_w(z/L)$

(Def of L: $L = -\frac{1}{k}\frac{u_*^3}{\overline{w'\theta'}_o}(\frac{g}{\overline{\theta}})^{-1}$)

→ $\lim_{-z/L \to \infty} \phi_w(z/L) \sim (-z/L)^{1/3}$



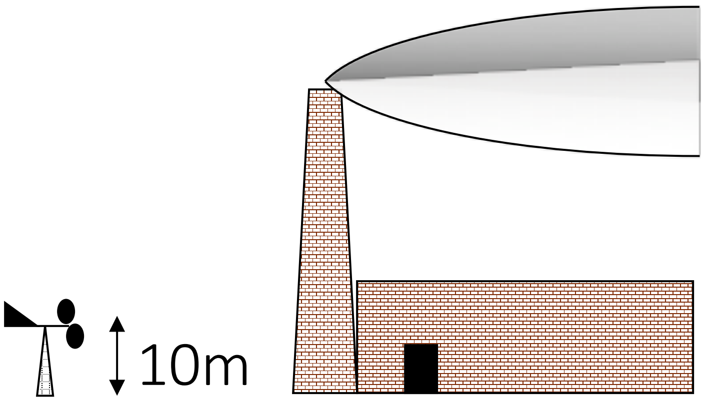
Applications: SL Scaling

→ determine wind velocity at different height than available

For example:

→ wind speed @ source height for dispersion modeling

→ numerical model (validation & assimilation)

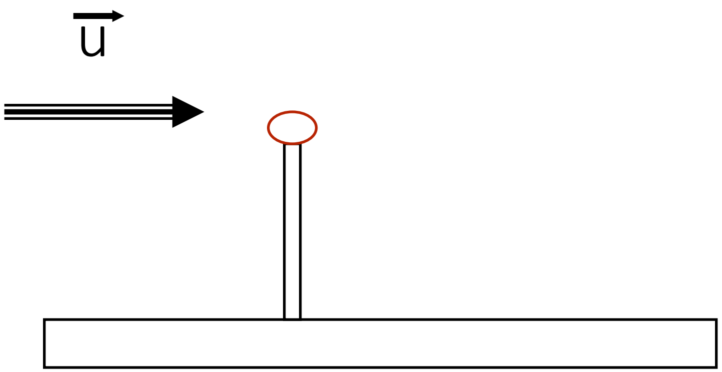


Applications: SL Scaling

$$\overline{u}(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) - \Psi_m(z/L)\right]$$

Wind profile Surface Layer

WMO standard: 10m



→ TAWES

for example:

→ want to use observation for dispersion modeling

→ stack height: 50 m

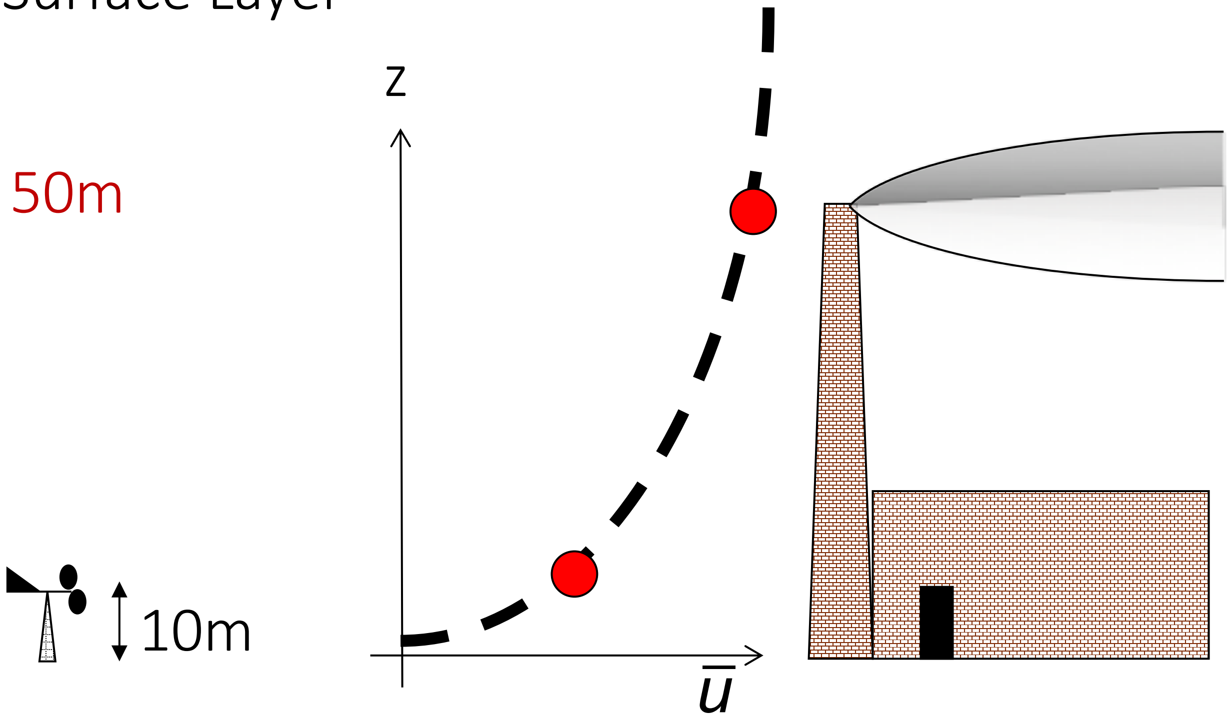
Applications: SL Scaling

$$\overline{u}(z) = \frac{u_*}{k}[\ln(\frac{z}{z_o}) - \Psi_m(z/L)]$$

Wind profile Surface Layer

stack height: 50m

Observation:
10m



Applications: SL Scaling

→ determine wind velocity at different height than available

→ determine turbulent (surface) fluxes from observation / or model value of mean wind speed

example: numerical model

→ available: mean wind speed at model level 1 (determined from solving cons. eq.)

→ coupling to surface: need the surface fluxes:

$$\overline{u'w'}_o, \overline{w'\theta'}_o$$

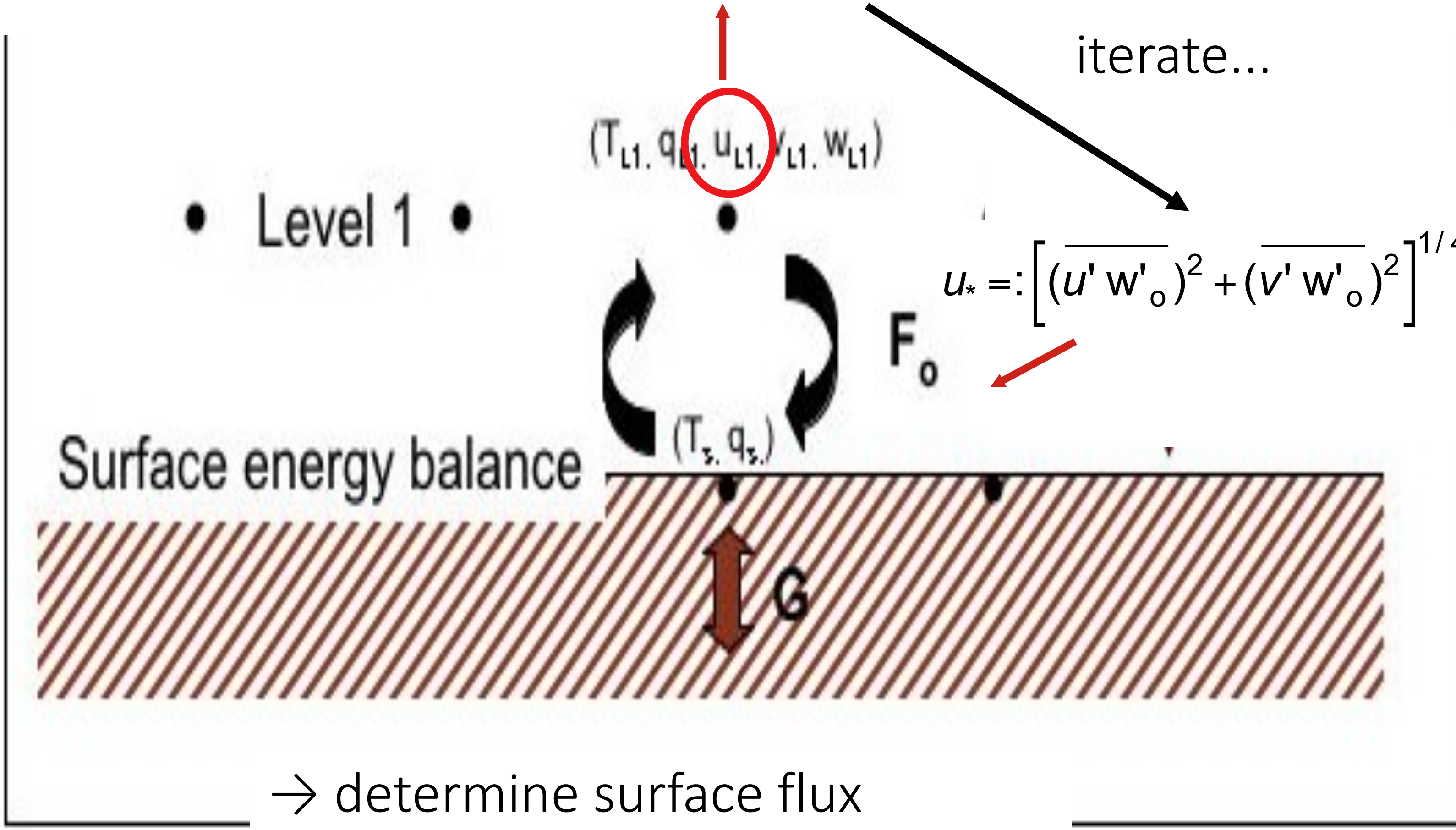
(surface exchange parameterization)

$$\overline{u}(z) = \frac{u_*}{k}\left[\ln\left(\frac{z}{z_o}\right) - \Psi_m(z/L)\right]$$

← z_o (input)

iterate...

$$u_* =: \left[\left(\overline{u'w'_o}\right)^2 + \left(\overline{v'w'_o}\right)^2 \right]^{1/4}$$



→ determine surface flux

→ coupling atmosphere - surface

Applications: SL Scaling

- → determine wind velocity at different height than available
- → determine turbulent (surface) fluxes from obs of mean wind speed (→ L, stability)
- → pollutant dispersion models
- → models for CO_2 exchange

Summary: SL Scaling

→ based on *surface fluxes:* $\overline{w'\theta'}_o \quad \overline{u'w'}_o$

→ *Surface Layer* = 'constant flux' layer

→ one π -group: z/L

→ every scaled mean variable: $\dfrac{\bar{a}}{a_*} = f_a(\pi_1)$

→ works for wind profile, temperature profile, specific humidity profile, vertical velocity variance, scalar variances, ...

→ works for spectra (chapter 7)

→ works for mean concentrations

→ does not work (very well): horizontal velocity variances