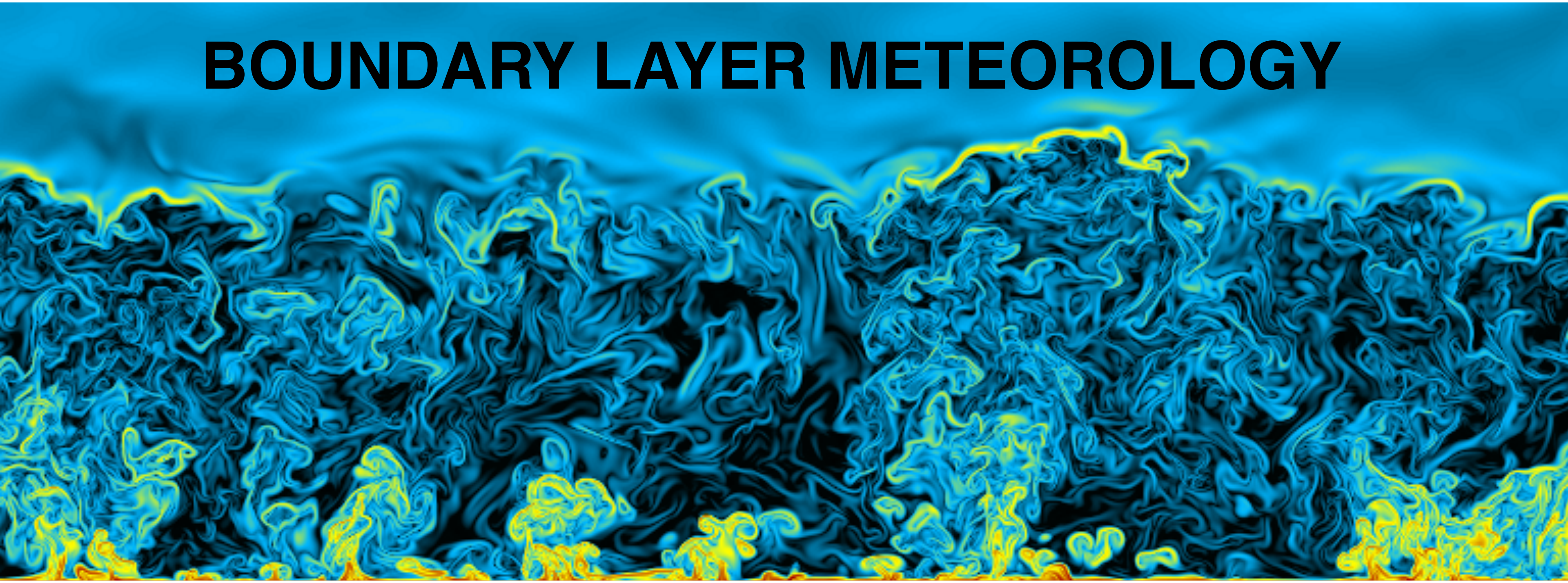
BOUNDARY LAYER METEOROLOGY



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Department of Atmospheric and Cryospheric Sciences

Chapter 3

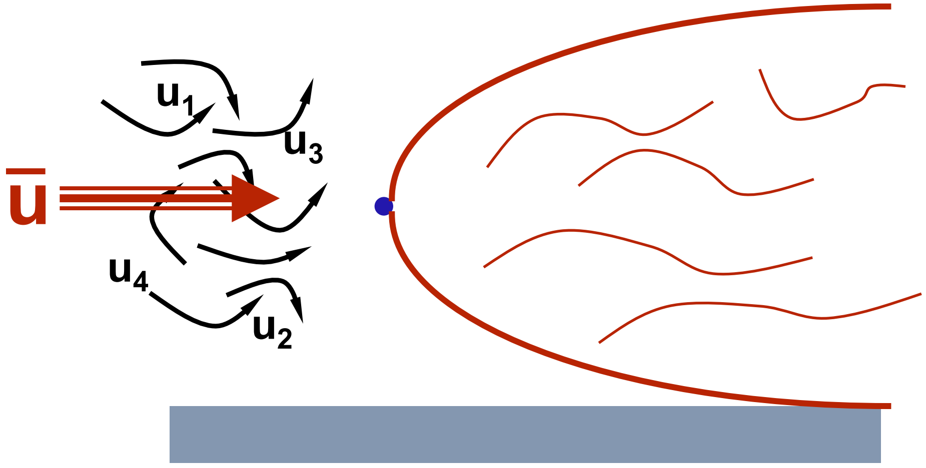
Statistical treatment of turbulence

Content

Motivation

Why do we use statistics when dealing with turbulence?

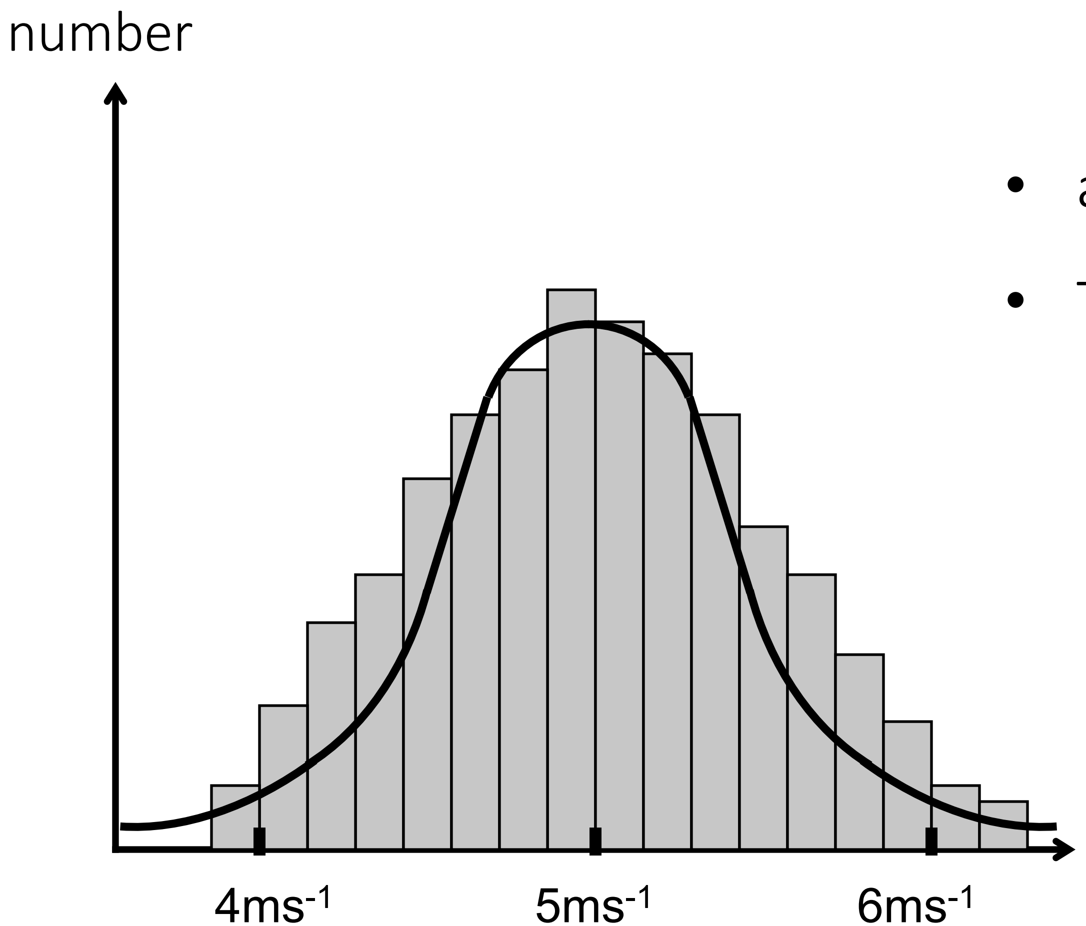
Motivation



- irregular, different each second…
- cannot / do not want: describe each trajectory
- statistical treatment!

Distributions

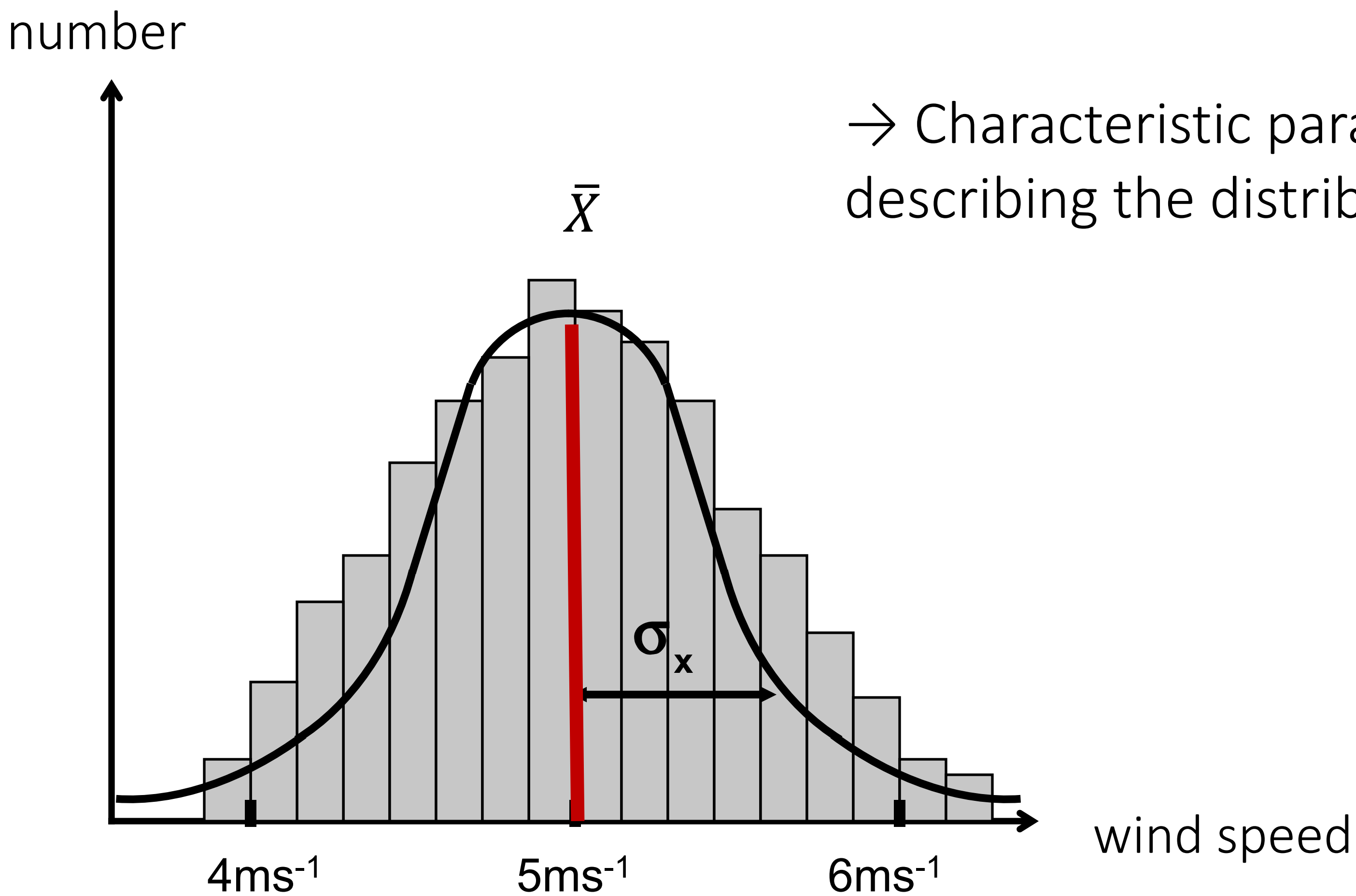
Represent measurements depending on their likelihood



- as bin size → 0
- Theoretical distribution

Distributions

→ Likelihood presented through: Probability Density Function(PDF, **P**)

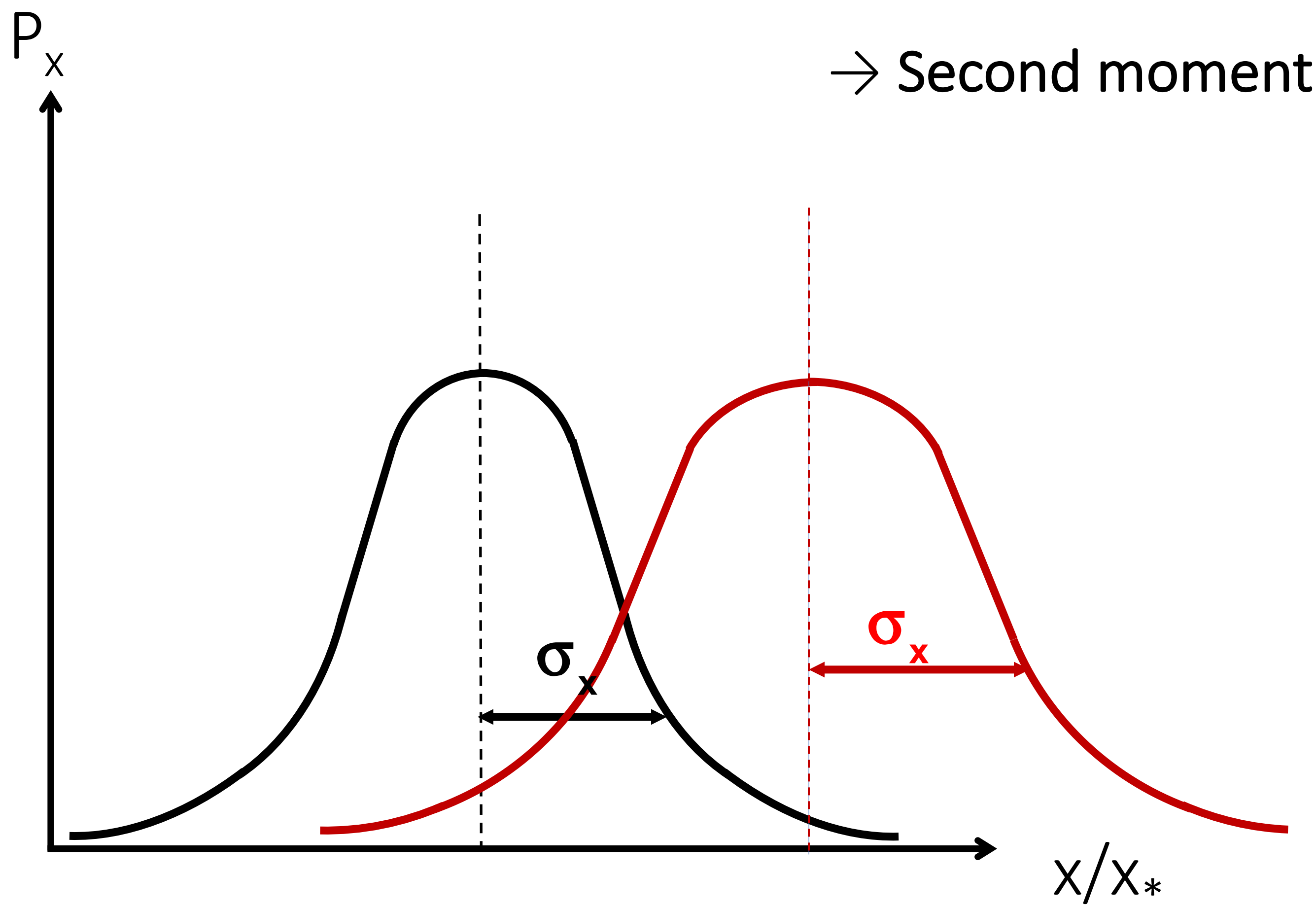


→ Characteristic parameters describing the distribution

Distributions - Variance

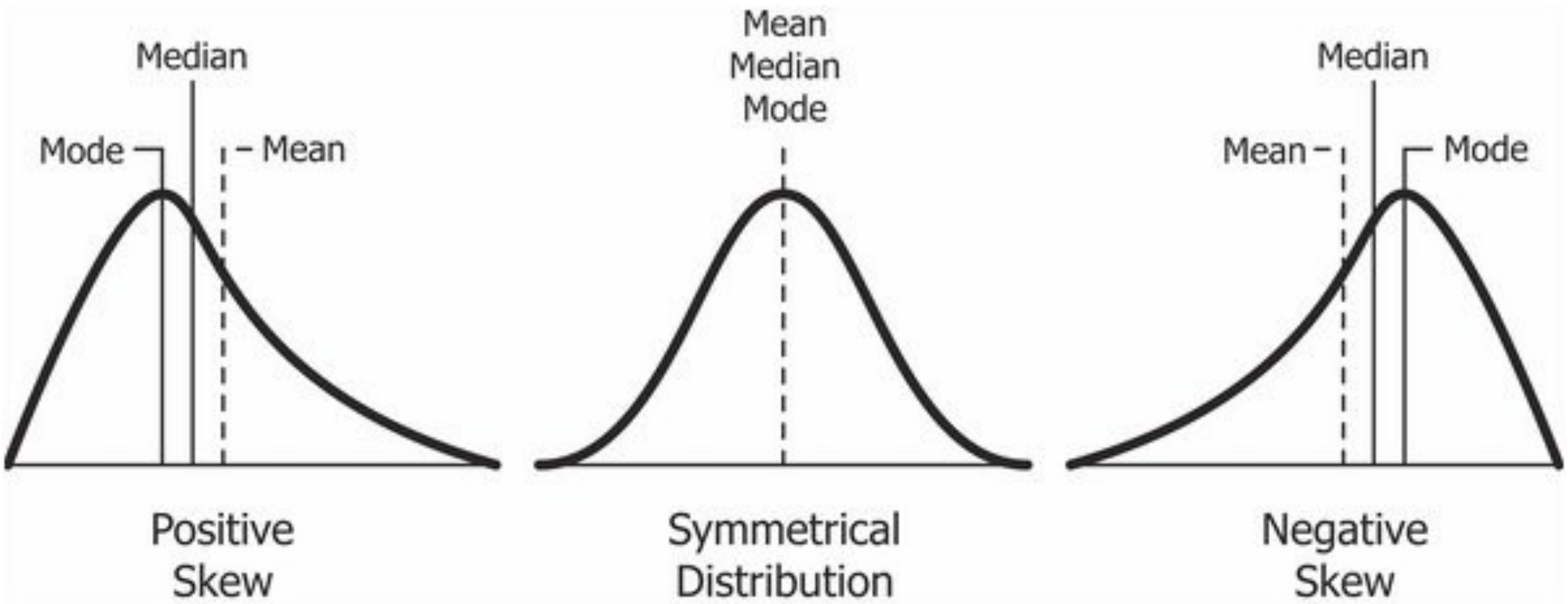
→ **First moment**: mean

→ **Second moment**: variance

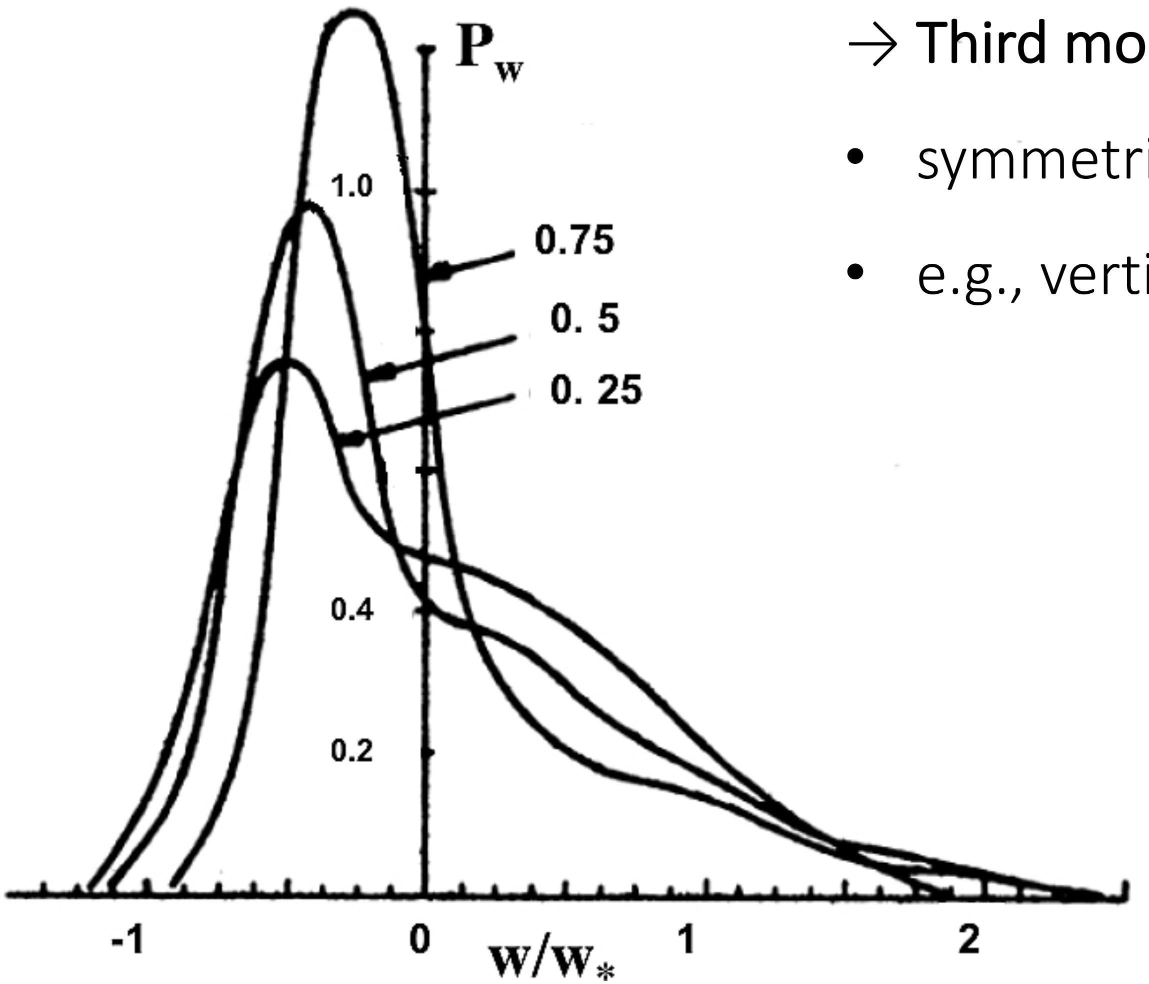


Distributions - Skewness

→ **Third moment**: Skewness



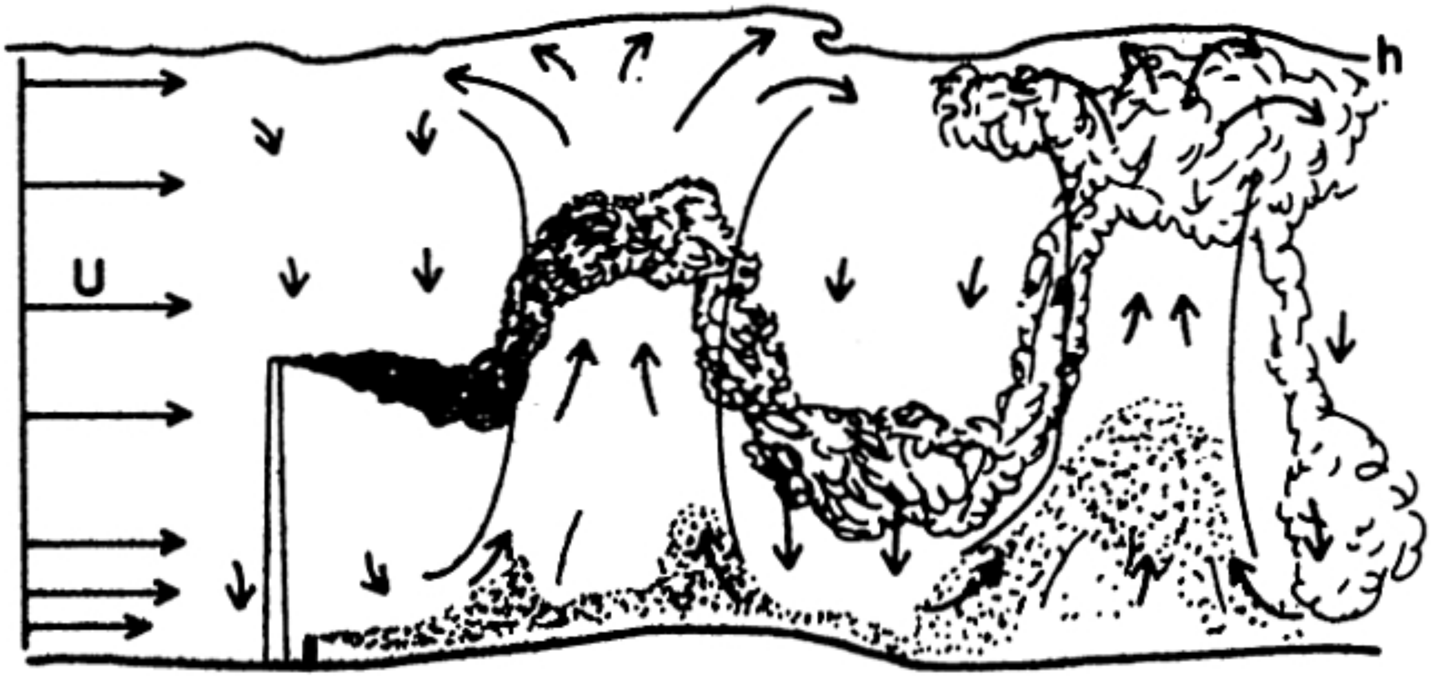
Distributions - Skewness



→ **Third moment**: Skewness

- symmetric?
- e.g., vertical velocity in CBL

CBL – skewness w

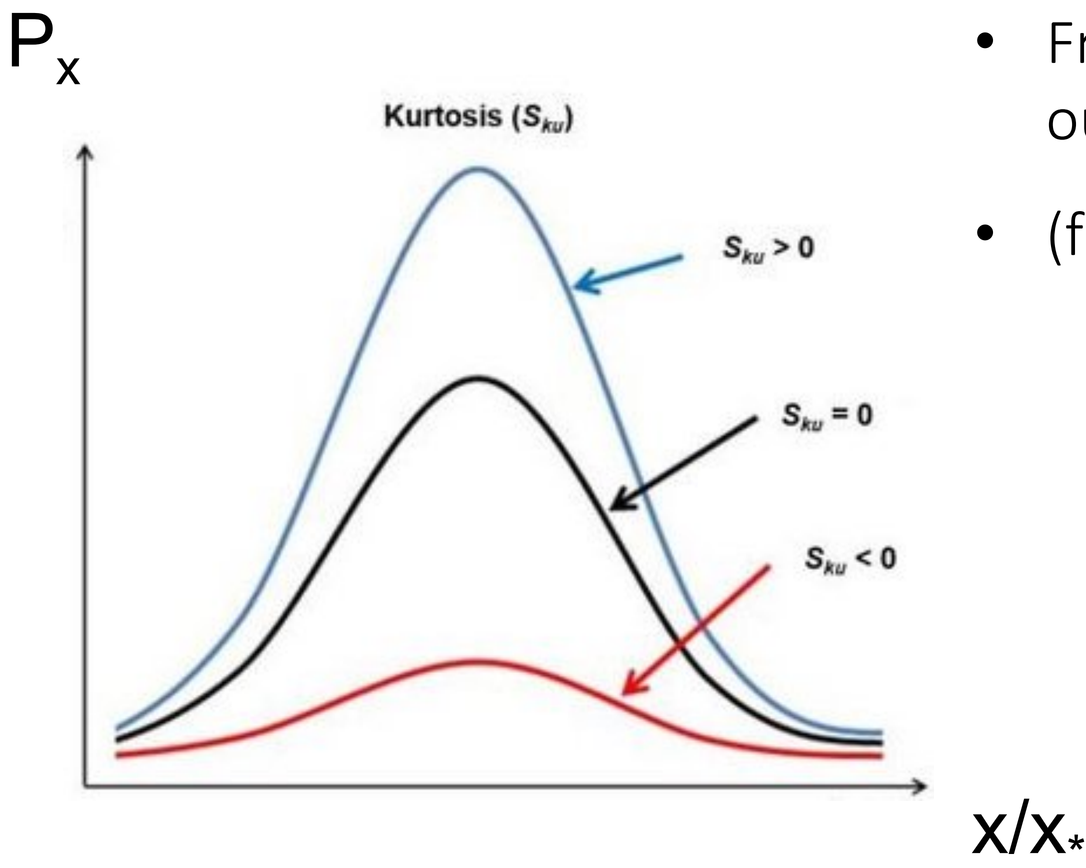


→ single 'thermals' with strong positive w
→ occupy about 30-40% of surface
→ median (w) slightly negative

Distributions - Kurtosis

→ **Fourth moment:** Kurtosis

- Frequency of extremes outliers
- (flatness of a distribution)



Probability density function

- completely characterized by its moments
- average:

$$\overline{a} = \int_{-\infty}^{\infty} a P_a da$$

- generally:

$$\overline{f(a)} = \int_{-\infty}^{\infty} f(a) P_a da$$

- moments:

$$\overline{a^n} = \int_{-\infty}^{\infty} a^n P_a da$$

- central moments:
  → n=0: norm
  → n=1: =0
  → n=2: variance
  → n=3: skewness

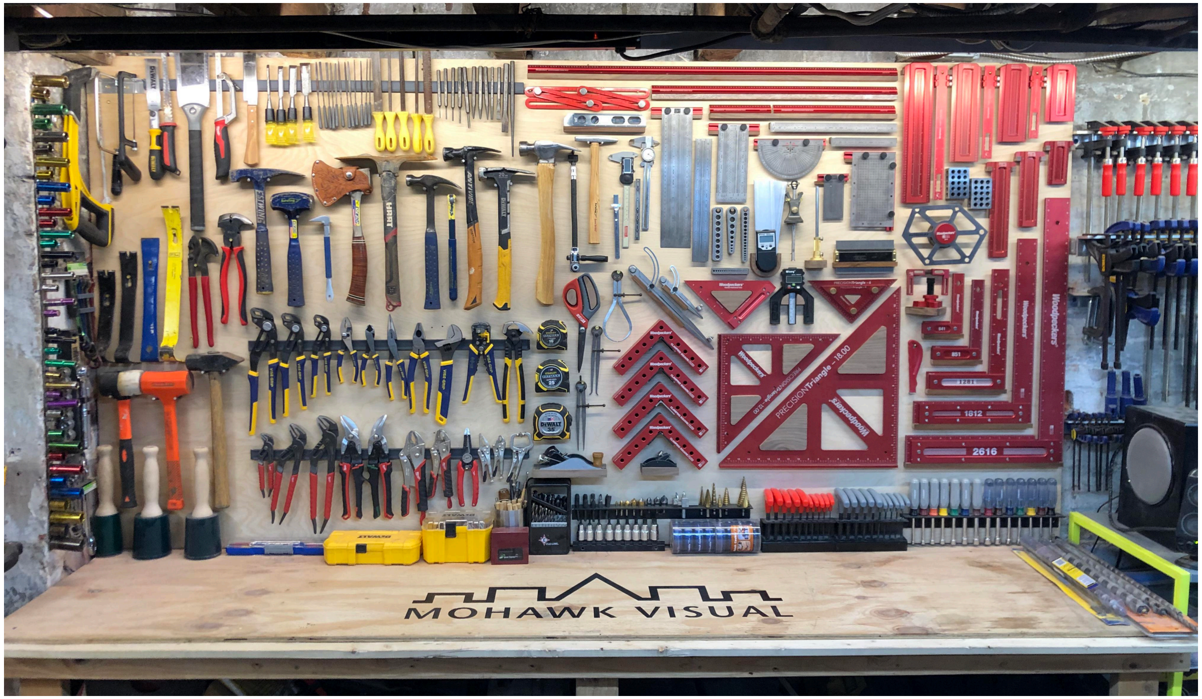
$$\overline{(a - \overline{a})^n} = \int_{-\infty}^{\infty} (a - \overline{a})^n P_a dx$$

Normal Distribution

$$P_a = \frac{1}{\sqrt{2\pi}\sigma_a} \exp\left\{ -\frac{1}{2} \frac{(a-\overline{a})^2}{\sigma_a^2} \right\}$$

Moments of the normal distribution
→ skewness = 0
→ kurtosis = 3
→ all higher moments = 0

Tools



1. Stationarity

Stationarity

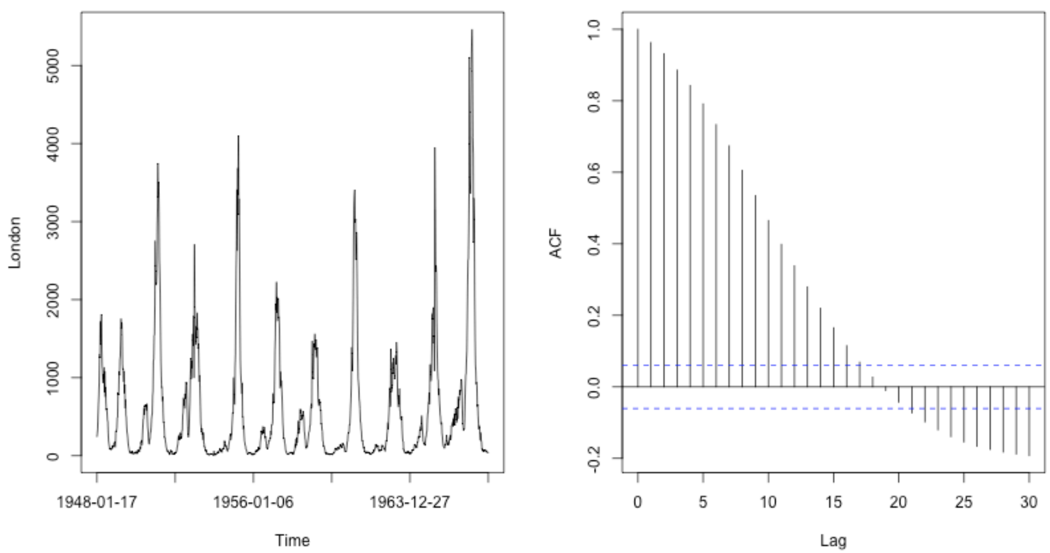
Consider: time correlation of a variable with itself

$$\overline{a(t)\cdot a(t')} =: C_a(\tau, T)$$

τ = t - t'

T= abs. time

Auto-covariance function



Stationary turbulence

stationary turbulence $\longleftrightarrow$ $\overline{a(t)\cdot a(t')} =: C_a(\tau)$

→ C_a independent of T

→ for all τ

→ in particular also for $\tau = 0$

→ $\overline{a(t)\cdot a(t)} =: C_a(0)$ = variance

Stationary turbulence

→ variance independent of T

order of stationarity:

→ correlation of the N-th order

$$\overline{a(t_1)\cdot a(t_2)\cdot a(t_3)\cdots a(t_N)} =: C_a^N(\tau_1,\tau_2,\tau_3,\ldots\tau_N)$$

→ independent of T

→ in particular: for $\tau = 0$

ALL moments of the distribution independent of T

in practice:

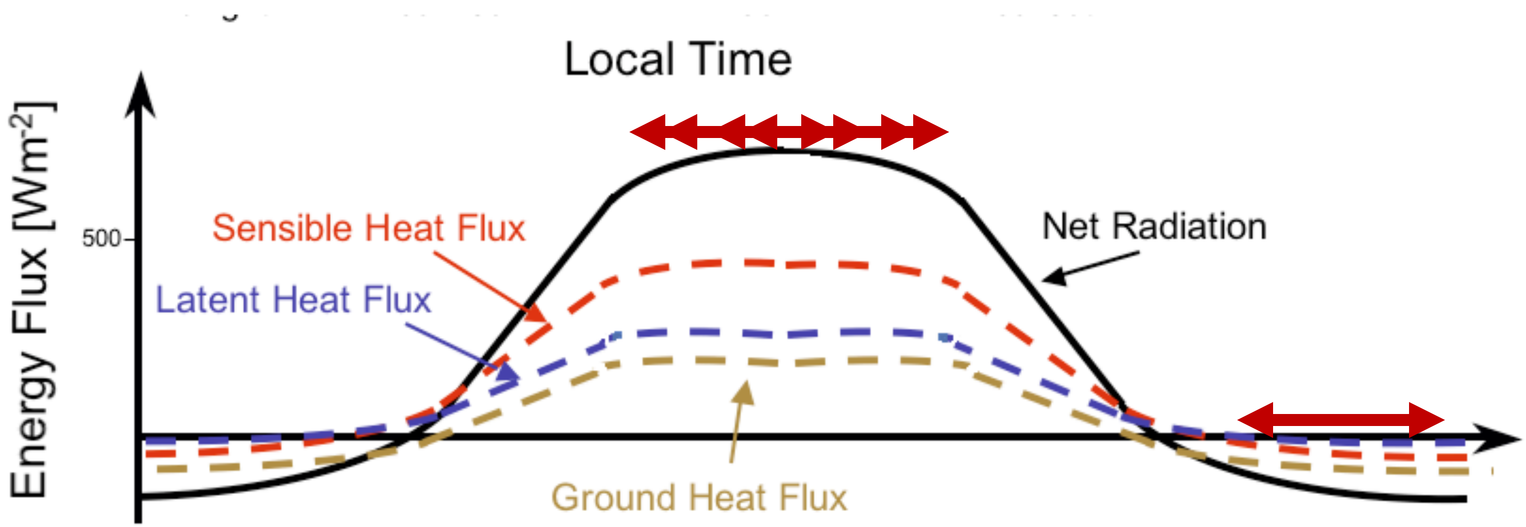
average, variance independent of T ‘enough’

Quasi - stationarity

- turbulence is **never** really stationary
  → turbulence is *dissipative*

2 time scales

T_f = 'forcing time scale': external processes
→ can be several hours



Quasi - stationarity

- Turbulence is **never** really stationary
  --> Turbulence is *dissipative*

2 time scales

T_f = 'forcing time scale': external processes

T_m = change of mean characteristics
→ how long until 'mean profile' has adapted to (external) change

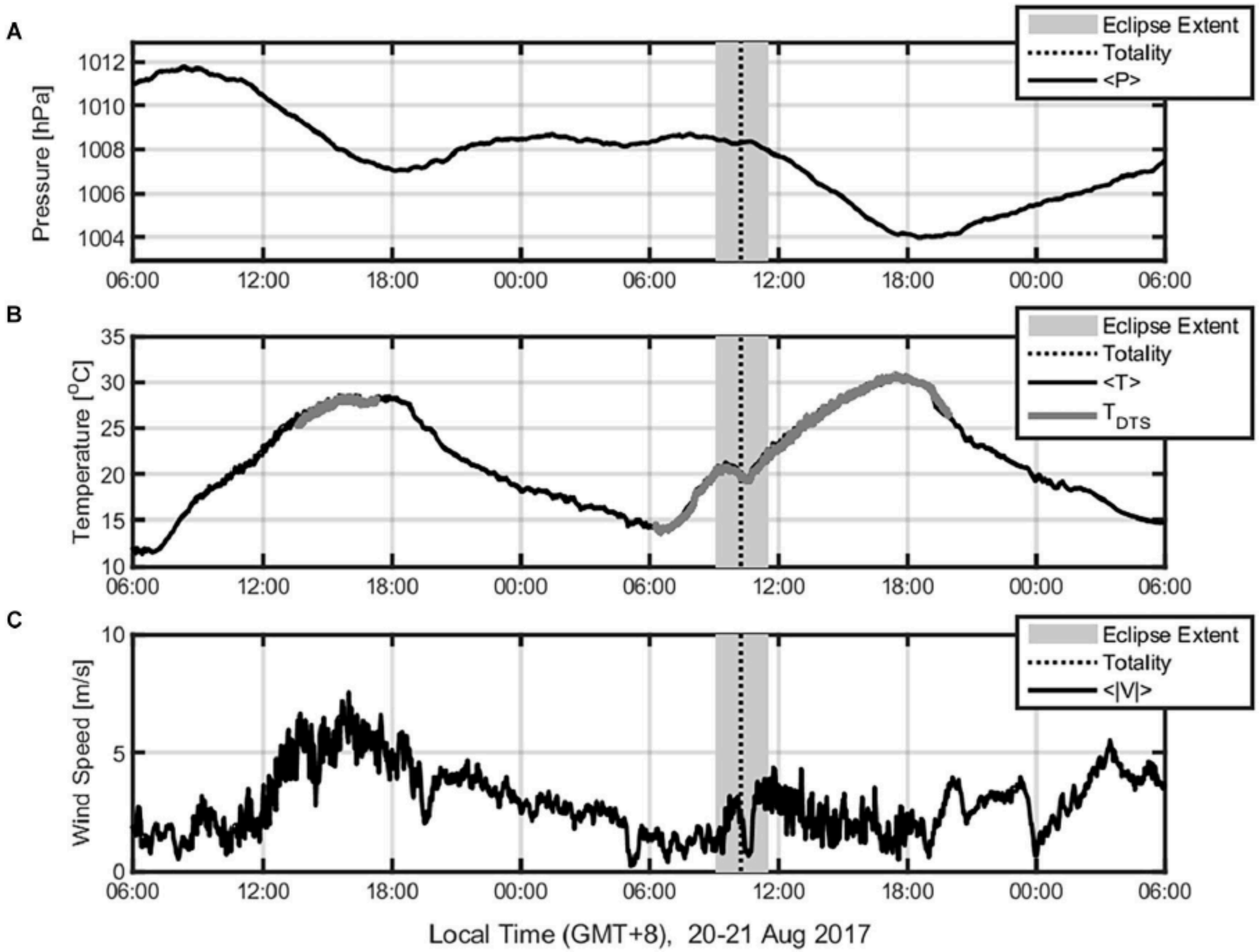
Quasi-stationarity: $T_m << T_f$

Example: Solar eclipse

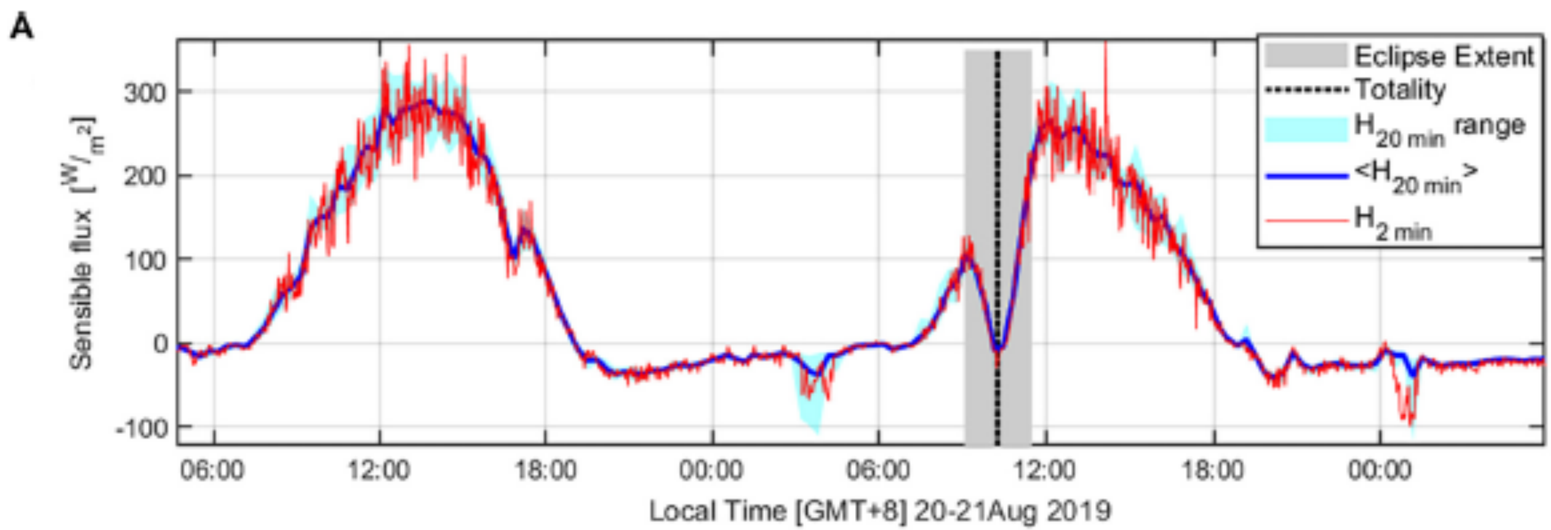
Higgins et al.2019: Ensemble-Averaging Resolves Rapid Atmospheric Response to the 2017 Total Solar Eclipse

https://www.frontiersin.org/articles/10.3389/feart.2019.00198/full

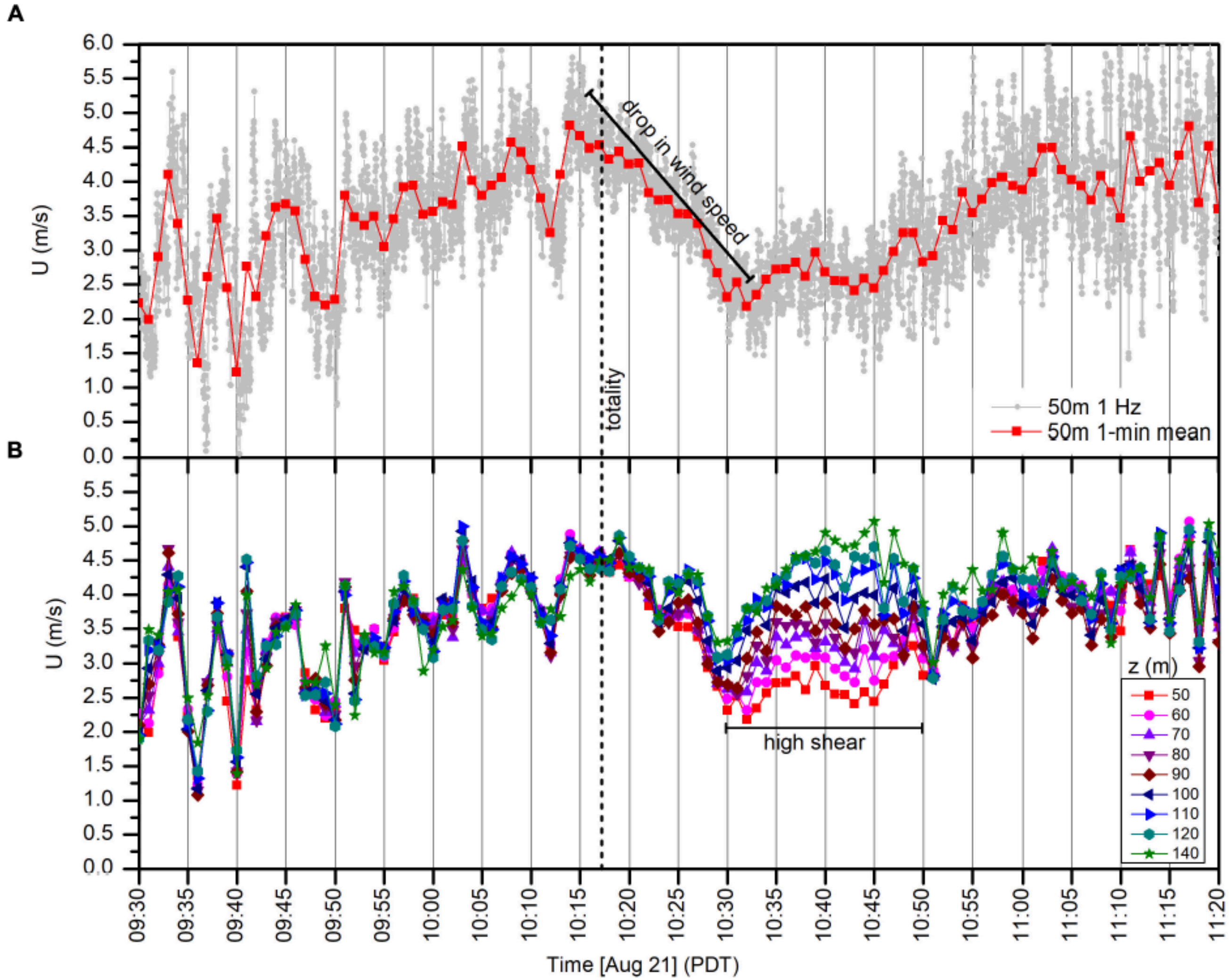
Example: Solar eclipse



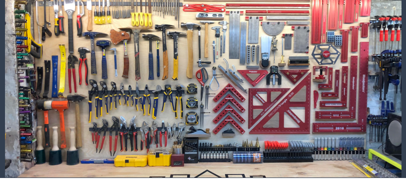
Example: Solar eclipse



Example: Solar eclipse



Tools



1. Stationarity
2. Homogeneity

→ analogue of stationarity in space

$$\overline{a(x)\cdot a(x')} =: C_a(\Delta x, \vec{r})$$

$\Delta x = x - x'$

$\vec{r}$ = position vector

→ homogeneous:

$$\overline{a(x)\cdot a(x')} =: C_a(\Delta x)$$

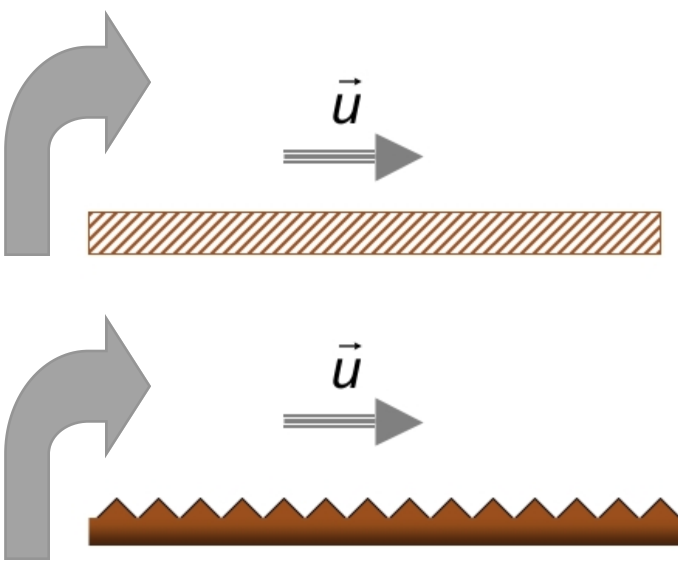
independent of $\vec{r}$

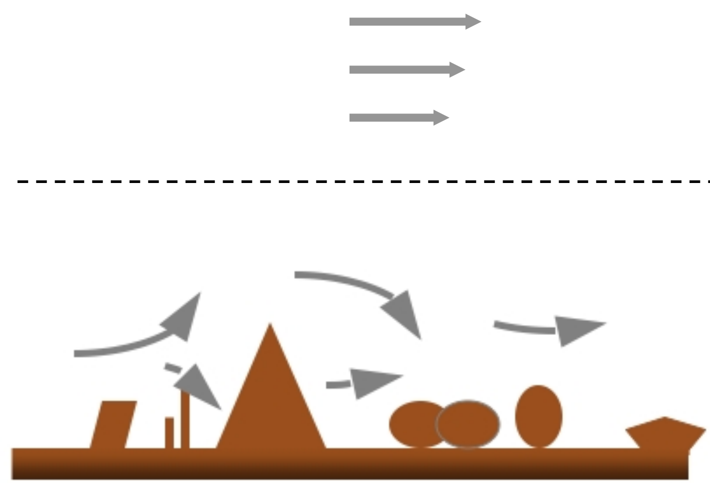
→ for all higher moments

→ in particular $\Delta x=0$

Homogeneity ⟷ Stationarity

homogeneity corresponds to stationarity in space (horiz)
→ if (sfc) forcing = const.



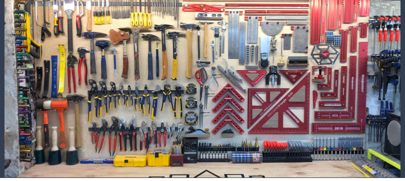


possibly stationary flow

no stationary flow (in Lagrangian framework)

stationary flow possible

Tools



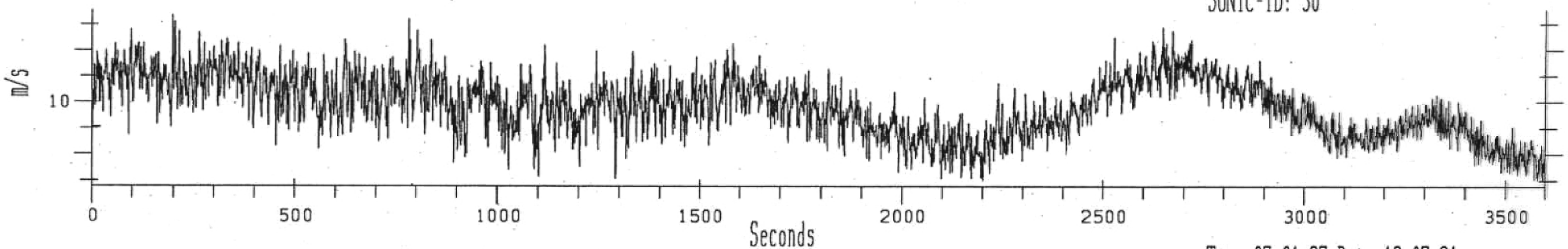
1. Stationarity
2. Homogeneity
3. Averaging

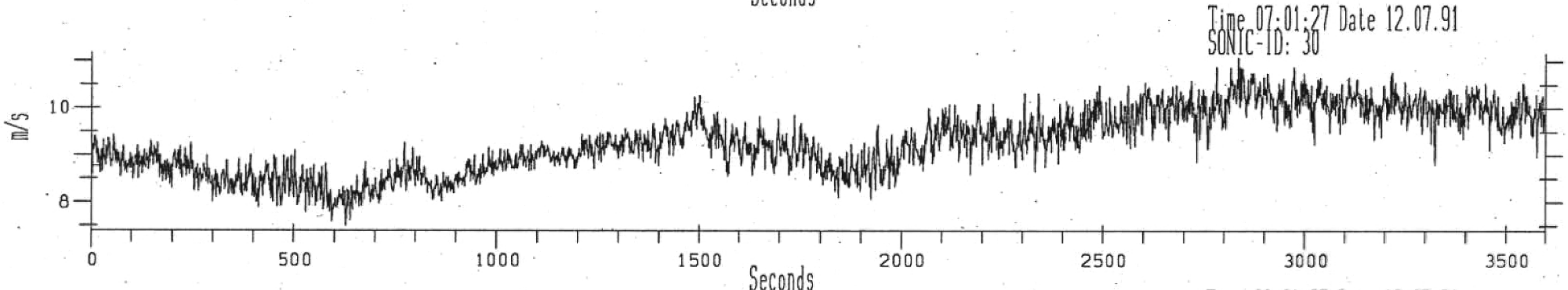
Averaging

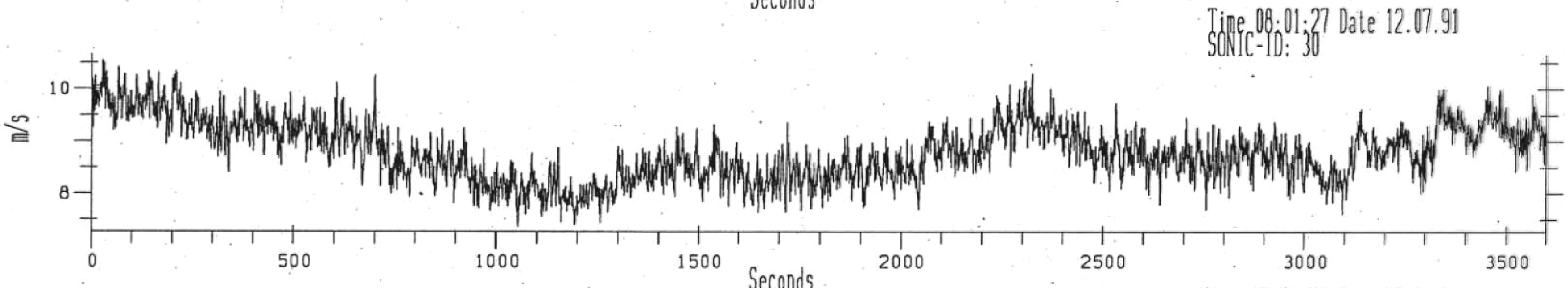
- goal (e.g. of a measurement):

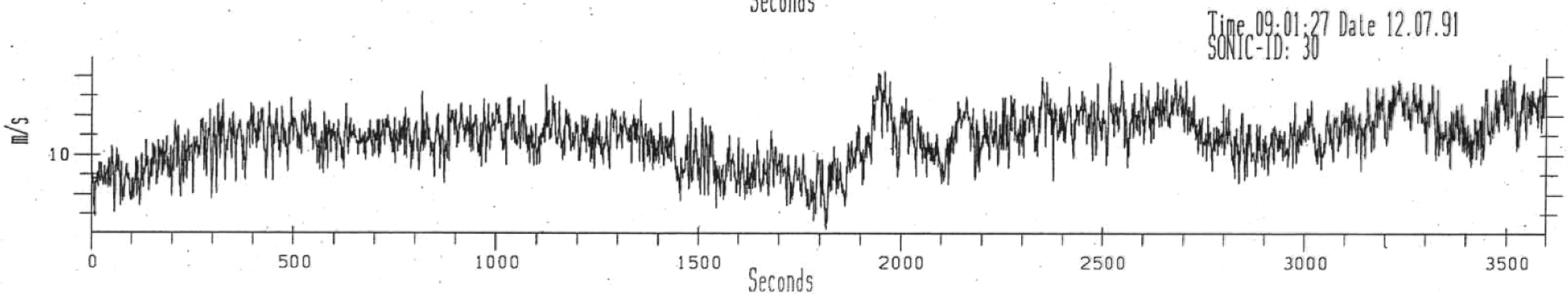
SONIC GREENLAND

Horizontal Windspeed









Averaging

- goal (e.g. of a measurement):
  → **not:** 'value' for this time at this place

  → would like: to learn something about the physical processes, which produce such a time series

  → to this time series, there is one 1m aside / '17 minutes' earlier, later, ...

  → the random process (producing *this* time series) both spatially as well as in the time domain *theoretically goes to infinity*

Ensemble averages

Ensemble = all possible realizations that can appear for a stationary process

→ average over all realizations:

$$\overline{a}^{e} = \frac{1}{N}\sum_{i=1}^{N} a_i(\vec{x},t) \quad \text{for} \quad a = a(\vec{x},t)$$

atmosphere: $N = \infty$

Ergodic Hypothesis

E.H: under certain conditions:

$$\overline{a}^{x} \rightarrow \overline{a}^{e}$$

$$\overline{a}^{t} \rightarrow \overline{a}^{e}$$

Wyngaard (2010):

‘The property that the time average of a stationary random variable and the space average of a homogeneous random variable converge to the ensemble average is called ergodicity’.

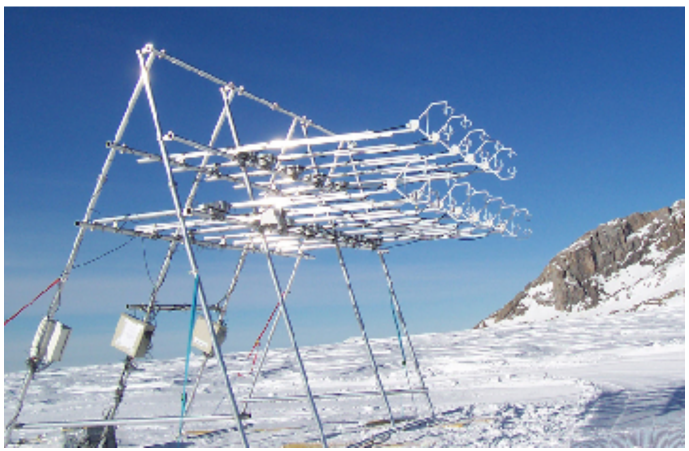
Ergodicity will always (implicitly) be assumed in real applications

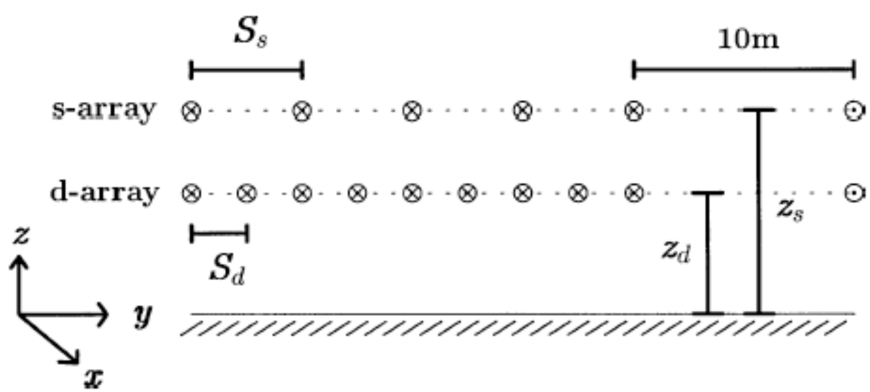
Averaging

Spatial average:

$$\overline{a}^x = \frac{1}{S} \iint_S a(\vec{x}, t) ds$$

→ want to know (structure of the turbulence)
→ difficult to obtain (lots of instruments!)



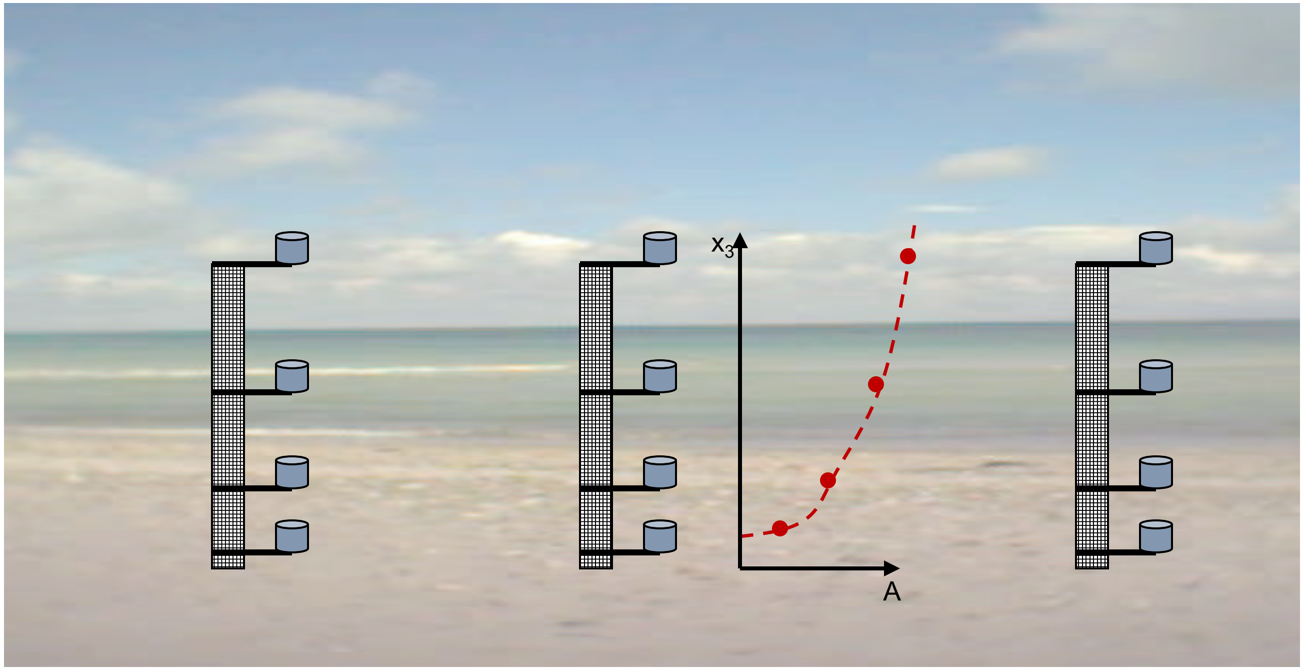


→ if horizontally homogeneous:
one characteristic profile!

HATS field campaign
→ Horst et al 2004
→ small-scale turbulence

Averaging

→ if horizontally homogeneous:
one characteristic profile!



Spatial Averaging

Fiber Optic Distributed Sensing (FODS)

Spatially Integrated Measurements

Averaging

spatial average: $$\overline{a}^{x} = \frac{1}{S} \iint_S a(\vec{x},t)\,ds$$

time average: $$\overline{a}^{t} = \frac{1}{T} \int_{t_1}^{t_1+T} a(\vec{x},t)\,dt$$

→ result of a measurement:

→ often: determine $\overline{a}^{t} \longrightarrow \overline{a}^{e}$ resp. $\overline{a}^{x}$

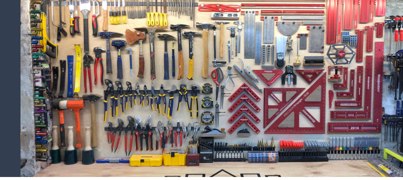
→ Ergodic Hypothesis!

Averaging Rules

Table 3.1: Useful rules for averaging

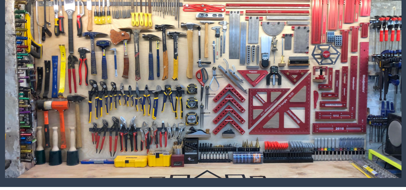
A, B are variables, *c* is a constant	
$\overline{c} = c$	
$\overline{(c \cdot A)} = c \cdot \overline{A}$	
$\overline{(\overline{A})} = \overline{A}$	An average behaves like a constant
$\overline{(\overline{A} \cdot B)} = \overline{A} \cdot \overline{B}$	
$\overline{(A \cdot B)} \neq \overline{A} \cdot \overline{B}$	The average of a product is not, in general, the product of the averages
$\overline{(A + B)} = \overline{A} + \overline{B}$	
$\overline{\left(\frac{\partial A}{\partial x}\right)} = \frac{\partial \overline{A}}{\partial x}$	This is an important property and derives from the Leibnitz theorem.

Intermediate Summary



- Pdf's
  → probability density function to describe the variables
  → fully characterized through its *moments*
- stationarity
  → all moments do not change with time
  → in practice: up to second moments enough
- homogeneity
  → is stationarity in space
- averaging
  → would need: average over all possible realizations
  → ensemble average
  → certain conditions: time/space average → ens. av.

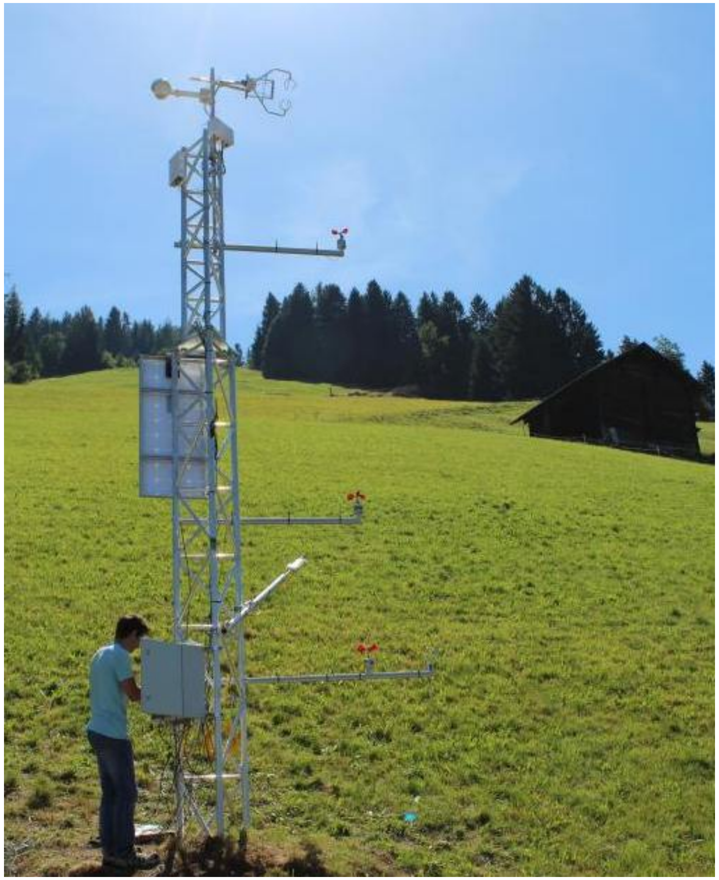
Tools



1. Stationarity
2. Homogeneity
3. Averaging
4. Taylor Hypothesis

Taylor Hypothesis

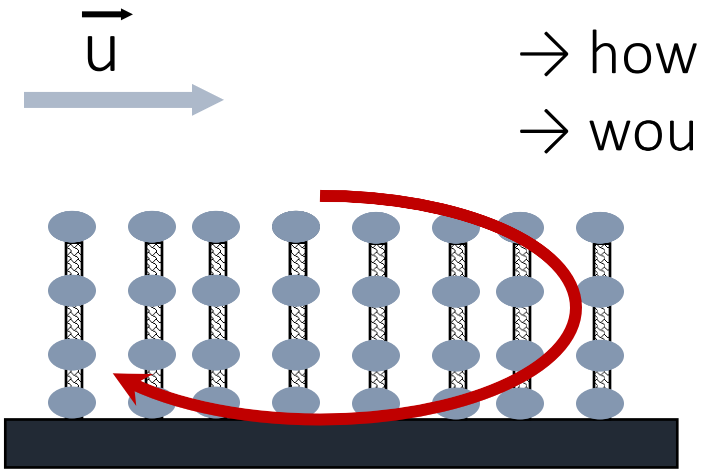
- Mostly: have time series (one instrument, i.e. place)



Taylor Hypothesis

- Mostly: have time series (one instrument, i.e. place)
  → want information on the *structure of turbulence*

More generally:



→ how can I observe 'an eddy'?
→ would need 1000's of instruments

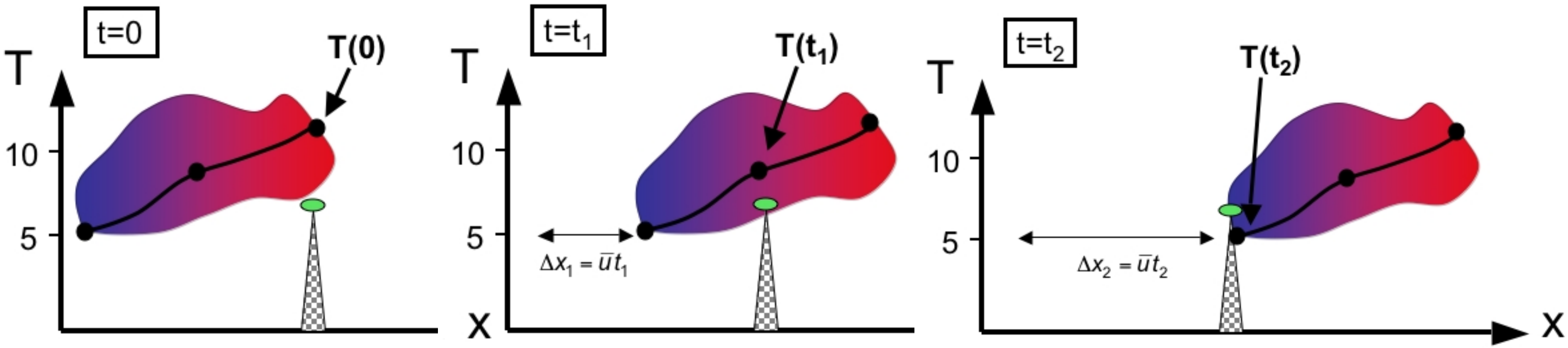
Taylor Hypothesis

Hypothesis:

The turbulence can be assumed to be **frozen** during the time it travels across the point of observation.

→ *Taylor's Frozen Turbulence Hypothesis*
→ *Geoffrey I. Taylor, 1938*

Taylor Hypothesis



Taylor Hypothesis

Hypothesis:

The turbulence can be assumed to be **frozen** during the time it travels across the point of observation.

→ applies, if:

$$T_f >> L_e / \overline{u}$$

→ process is stationary
→ T_f = forcing time scale
→ L_e = characteristic length
→ u = average wind speed

→ in practice:

$$\sigma_u / \overline{u} < 0.5$$

→ σ_u = measure of activity of turbulence
→ $\overline{u}$ = measure of advection

Taylor Hypothesis

mathematically: $D\zeta / Dt = 0$

$$\frac{D\zeta}{Dt} = \frac{\partial\zeta}{\partial t} + \frac{\partial\zeta}{\partial x}\underbrace{\frac{\partial x}{\partial t}}_{u} + \frac{\partial\zeta}{\partial y}\underbrace{\frac{\partial y}{\partial t}}_{v} + \frac{\partial\zeta}{\partial z}\underbrace{\frac{\partial z}{\partial t}}_{w} = 0$$

'frozen'

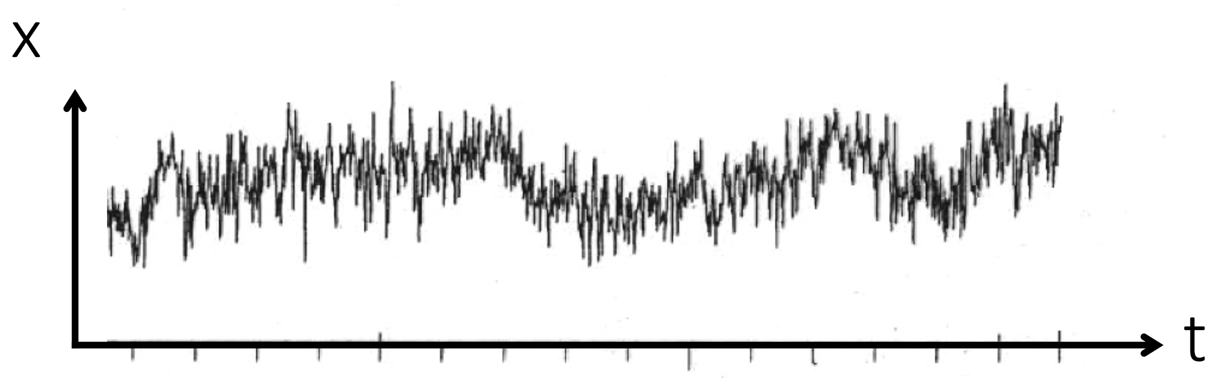
$$\boxed{\frac{\partial\zeta}{\partial t} = -\vec{v} \cdot \nabla\zeta}$$

'conservative flow field'

→ measured change in time corresponds to advected spatial structure

Reynolds Decomposition

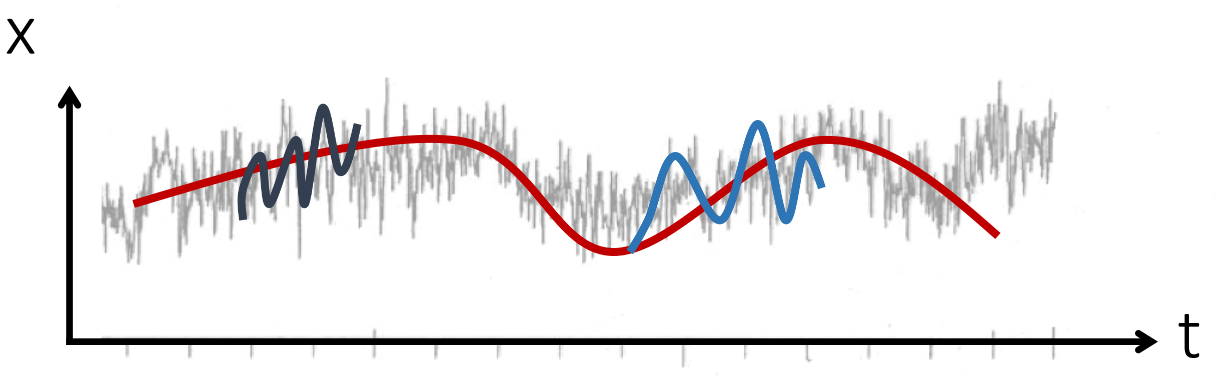
- averaging



→ is it now turbulence?
→ or average flow?
→ averaging over what time?
→ where does turbulence remain after averaging?

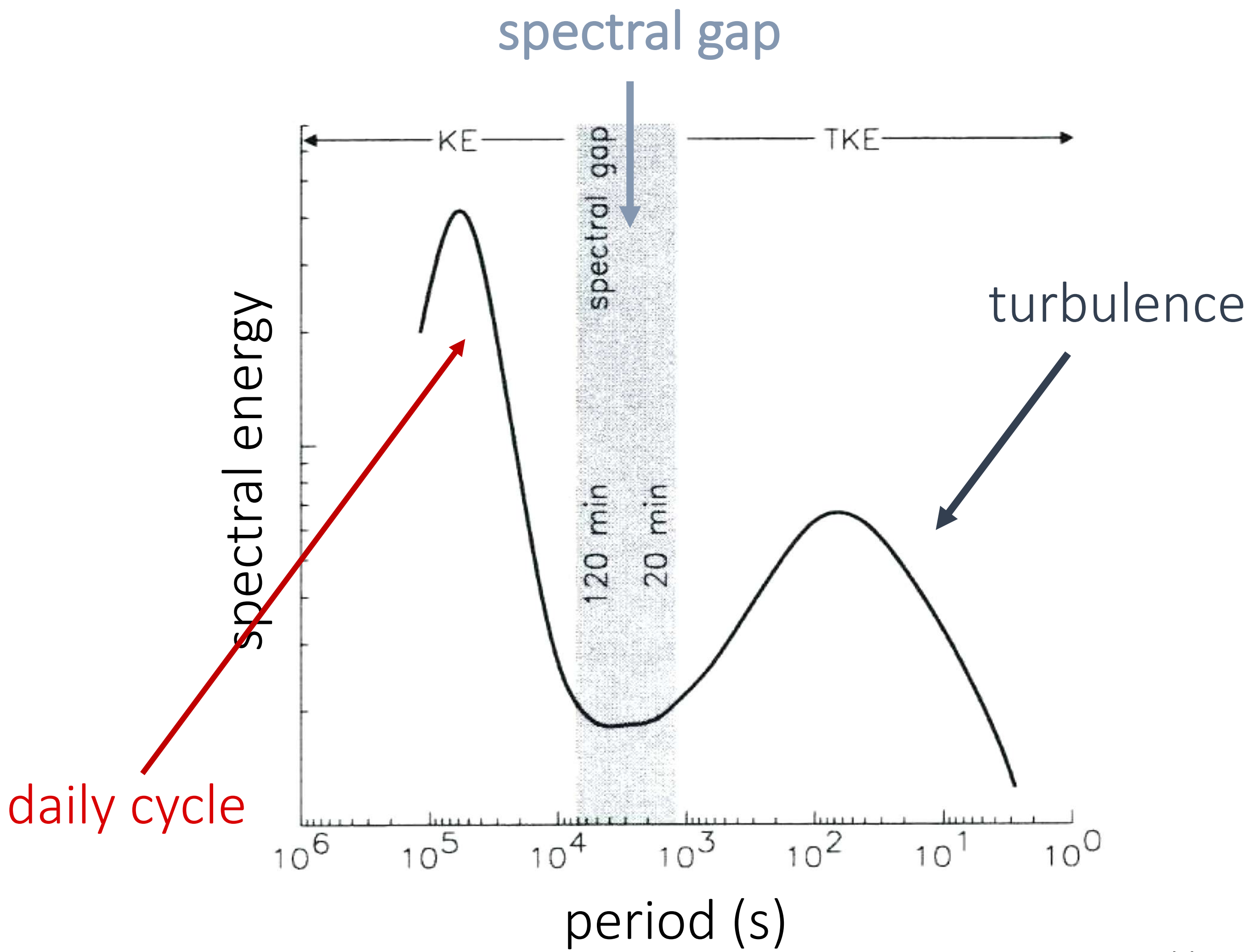
Reynolds Decomposition

- fluctuations with different periodicity
  → seasonal cycle, daily cycle, fast fluctuations



- spectral distribution
  → how much 'power' in which periodicity?
  → (see later, chapter 7)

(Idealized) Energy Spectra



Van Gorsel (2004)

Reynolds Decomposition

- pragmatic choice:
  → fluctuations faster than about 1h: turbulence
  → longer: average flow

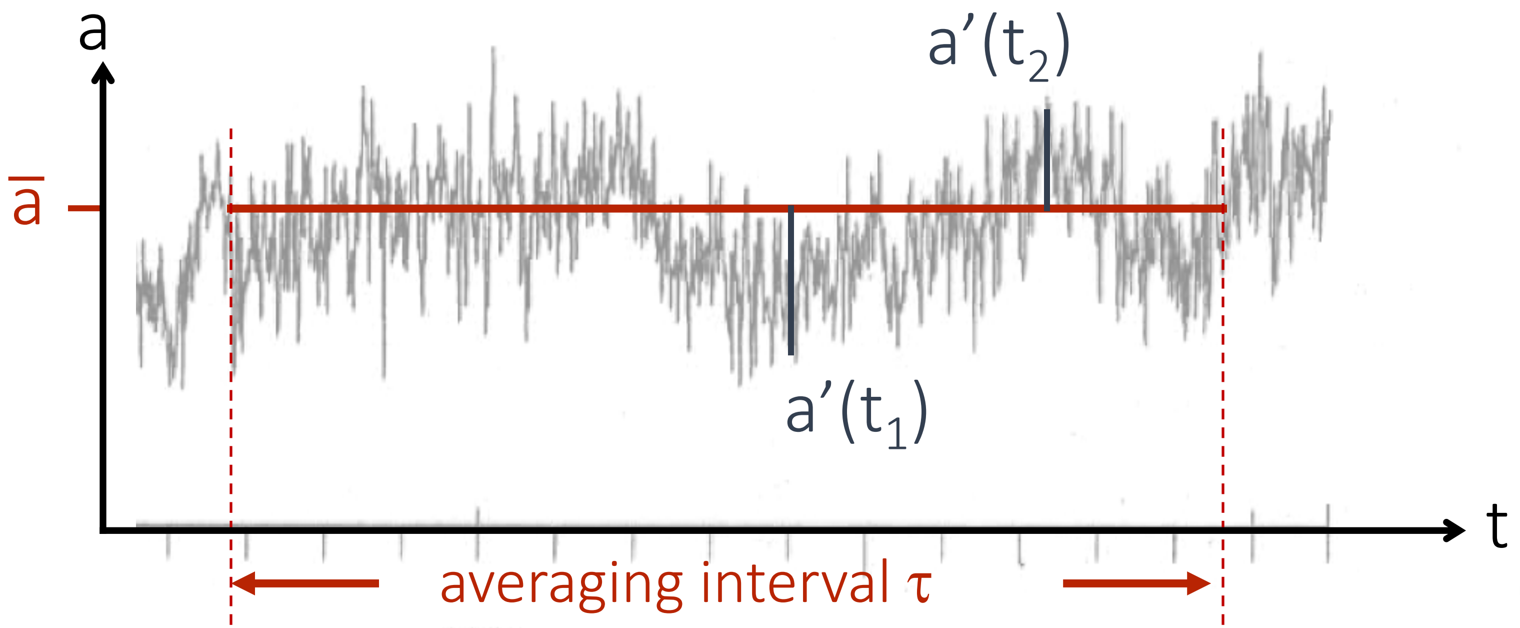
→ each variable a: $$a = \overline{a} + a'$$

total signal — average — fluctuation

$\overline{a}$ = time average

actually: ensemble average

Reynolds Decomposition



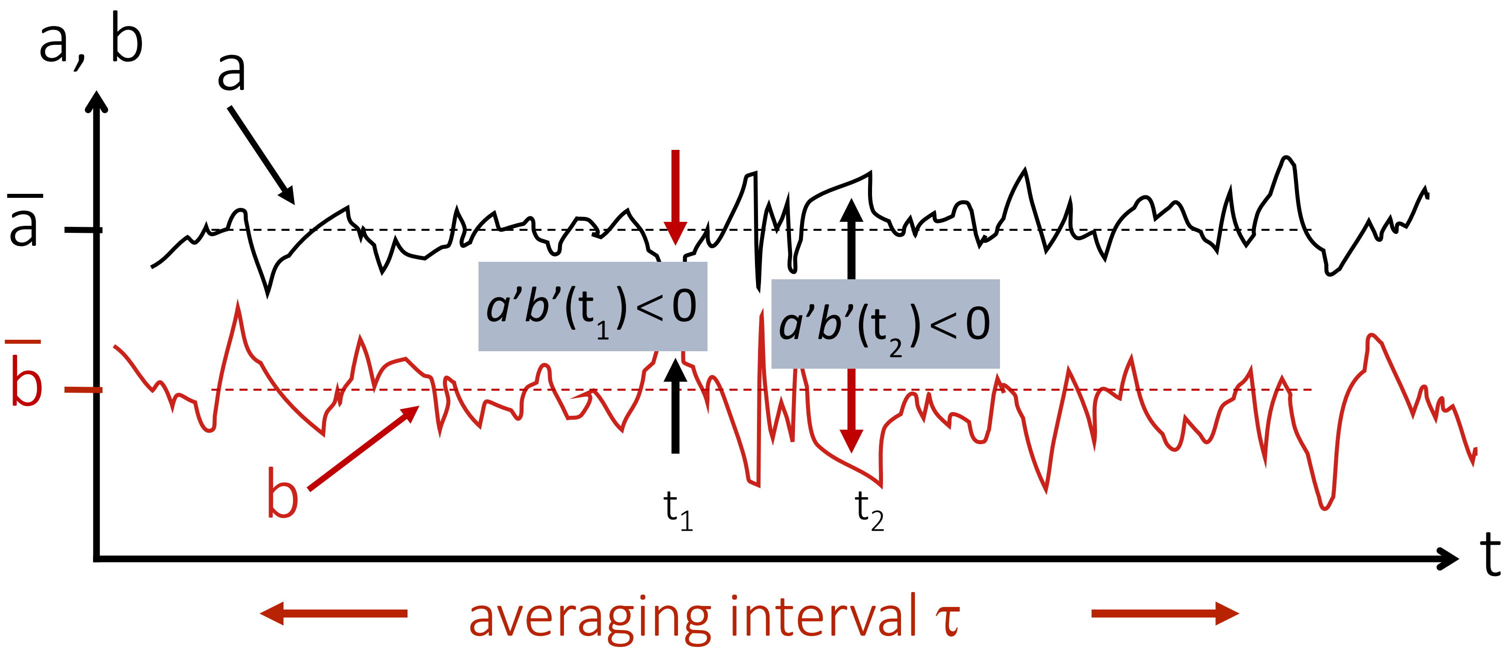
in practice:

- measure a(t): time resolution big enough (how to choose it?)
- compute: $\overline{a}$ for each averaging period
- from there: a'(t): for each measurement (20Hz)

Computation Rules for Reynolds Decomposition

	Table 3.2: Calculus for Reynolds Decomposition	
	a and b are variables, for which: $a = \overline{a} + a'$; $b = \overline{b} + b'$	
1)	$\overline{a'} = 0$	By definition
2)	$\overline{(\overline{a})} = \overline{(\overline{a} + a')} = \overline{a}$	By definition and 1)
3)	$\overline{(\overline{b} \cdot a')} = \overline{b} \cdot \overline{a'} = 0$	The average of a product involving a primed variable vanishes
4)	$\overline{(a \cdot b)} = \overline{(\overline{a} + a') \cdot (\overline{b} + b')}$ $= \overline{a} \cdot \overline{b} + \overline{a'b'}$	The covariance is not necessarily zero
5)	$\overline{a^2} = \overline{a}^2 + \overline{a'^2}$	The second term on the rhs corresponds to the Second central moment, i.e. the variance

Meaning of covariances

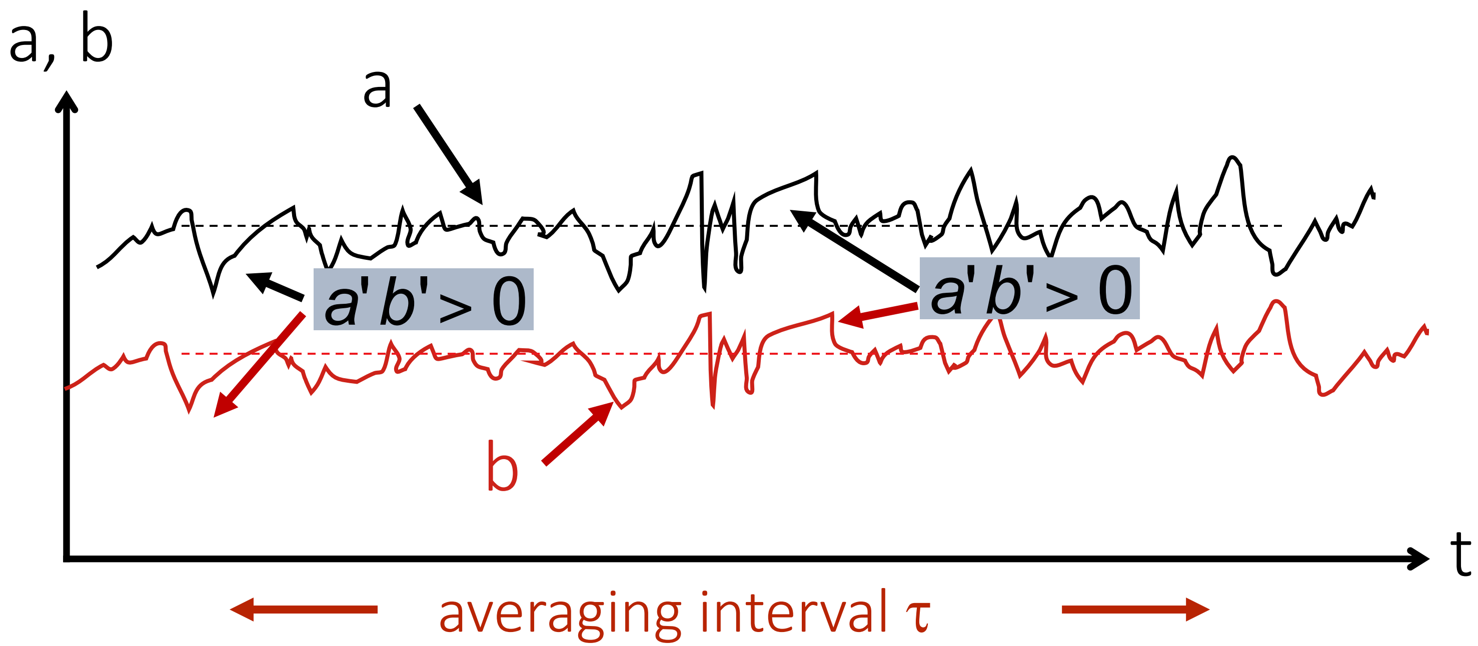


→ opposite behavior...

→ on **average** (here):

$$\rightarrow \overline{a'b'} < 0$$

Meaning of covariances

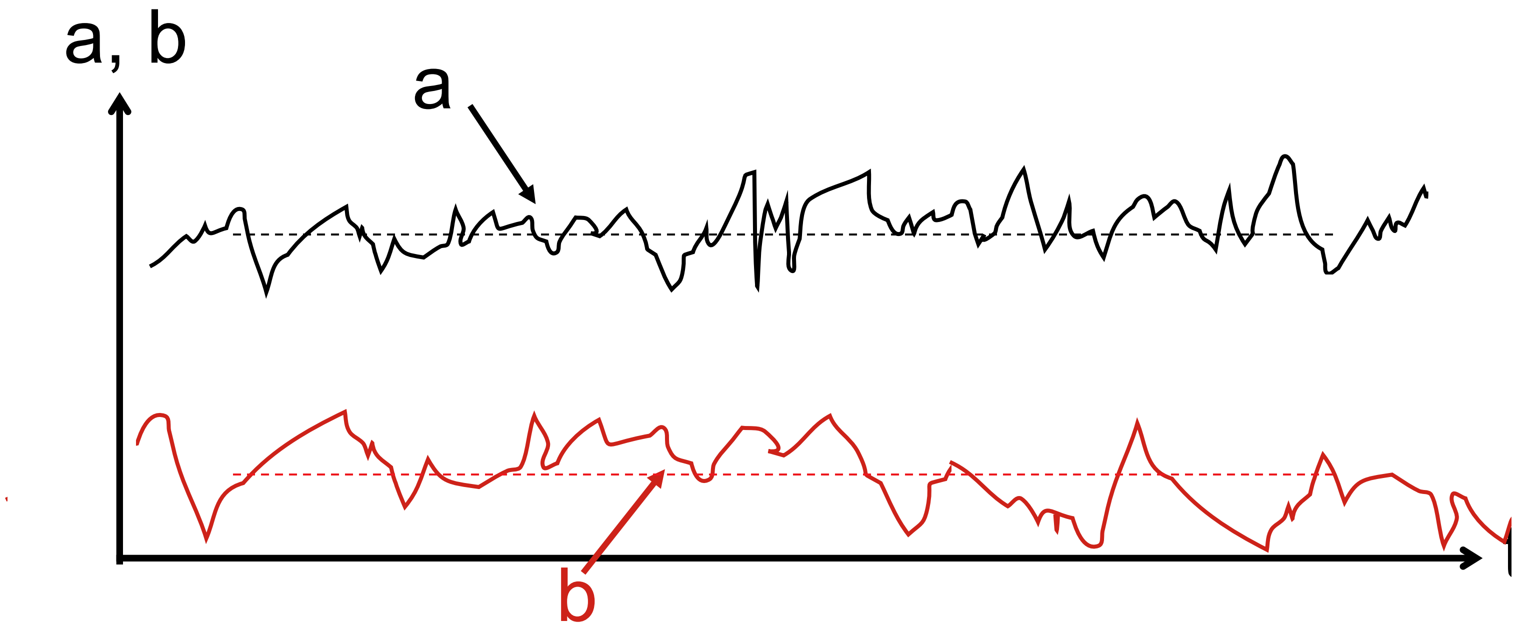


→ similar behavior....

→ on **average** (here):

→ $\overline{a'b'} > 0$

Meaning of covariances



→ a,b poorly correlated

→ $\overline{a'b'} \approx 0$

Meaning of covariances

- physical meaning
  → turbulent transport
- in general:
  → consider: physical description of transport

Description of Transport

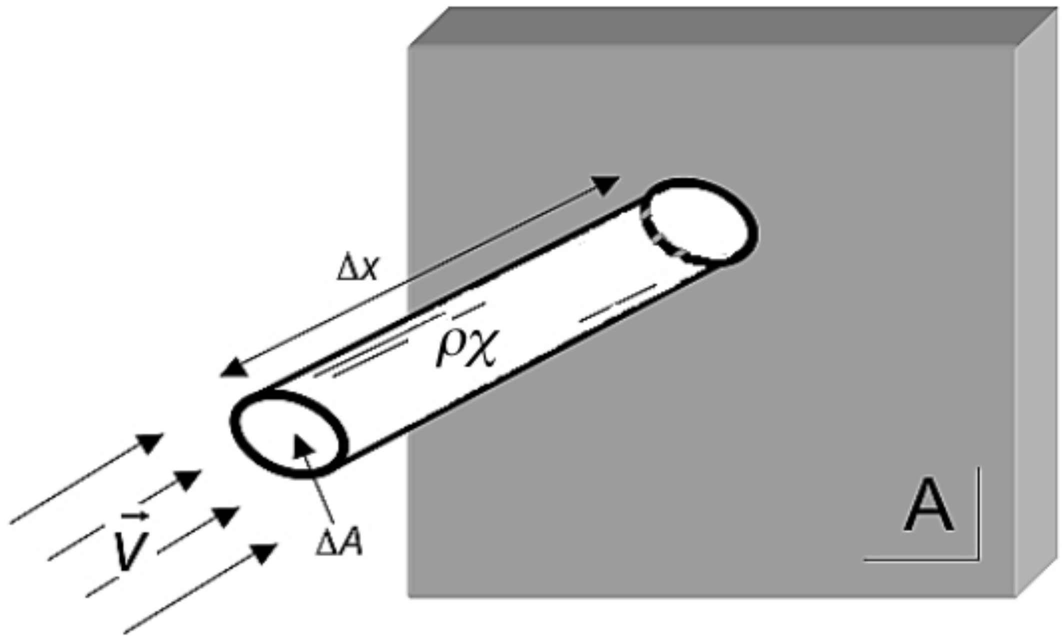
let: X = additive quantity ('countable')

then $$\mathrm{X} = \iiint_V \rho\chi dV$$ $[\mathrm{X}] = \mathrm{x}$

→ χ = specific quantity $[\chi] = \mathrm{x/kg}$

Description of Transport

transport of
X through the
surface ΔA :



$$F_X = \frac{X}{\Delta A \Delta t} = \frac{\rho\chi V}{\Delta A \Delta t} = \frac{\rho\chi \Delta A \Delta x}{\Delta A \Delta t} = \rho\chi \frac{\Delta x}{\Delta t}$$

→ infinitesimal:

$$\vec{F}_\chi = \rho\chi\vec{V}$$

- 3d
- vector quantity

Description of Transport

→ infinitesimal: $$\vec{F}_\chi = \rho\chi\vec{v}$$

→ Flux = ρ density of fluid
x specific transported quantity
v transport velocity

→ co-variances: $\overline{a'b'}$

→ often: a or b is a velocity component

→ for example: co-variance: $\overline{a'w'}$

→ transport of 'a' in vertical direction

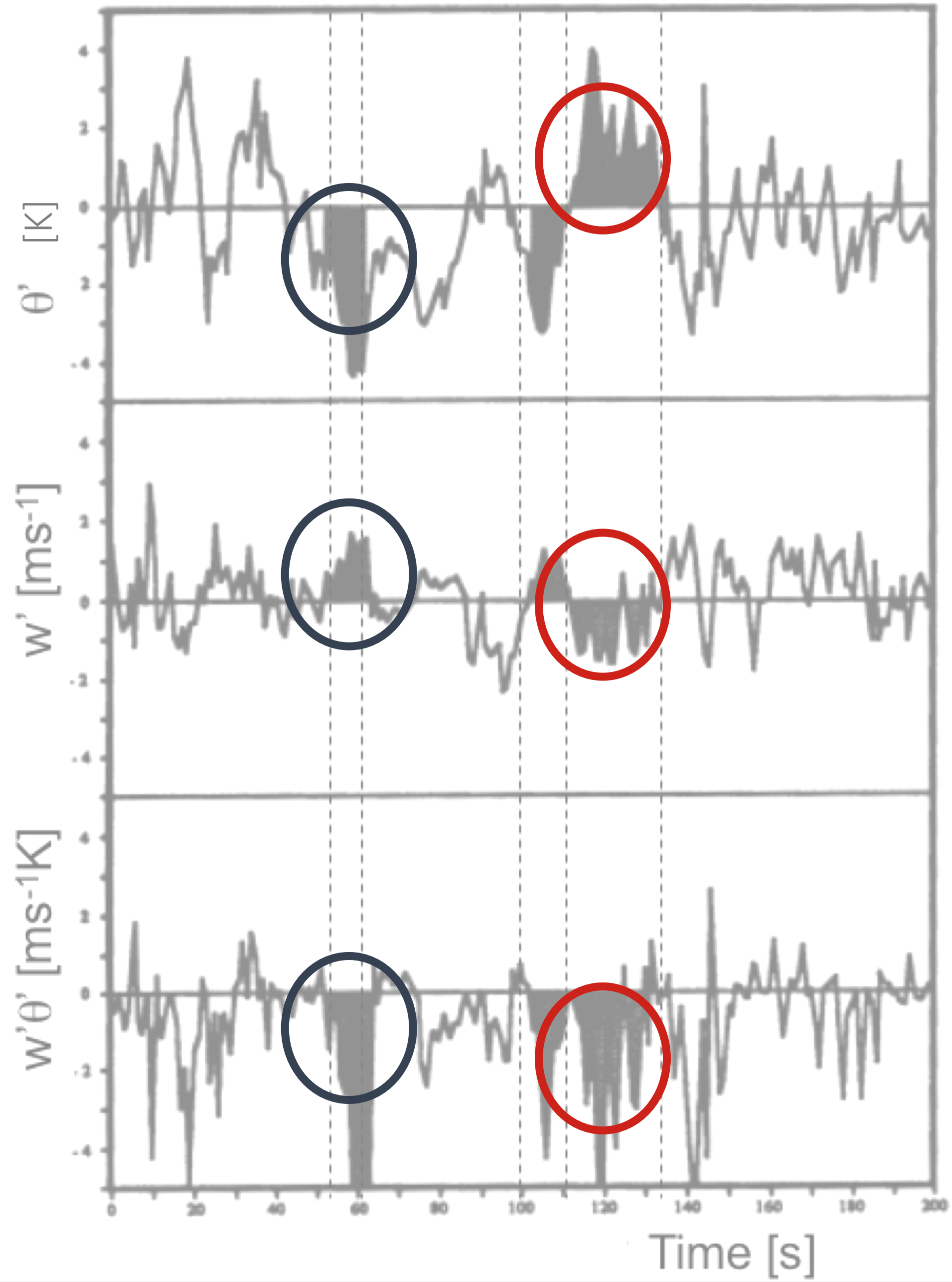
Covariances = Turbulent Transport

- these co-variances will be important:

$$\overline{w'\theta'},\ \overline{w'q'},\ \overline{u'w'},\ (\overline{v'w'})$$

- with: w' and a scalar (θ' , q'), resp. w' and u'
  → expresses transport of scalar quantities in **vertical** direction

- each instantaneous value, e.g. :
  → instantaneous flux (transport) $w'\theta'$
  → average: random or systematic, e.g.: $\overline{w'\theta'}$

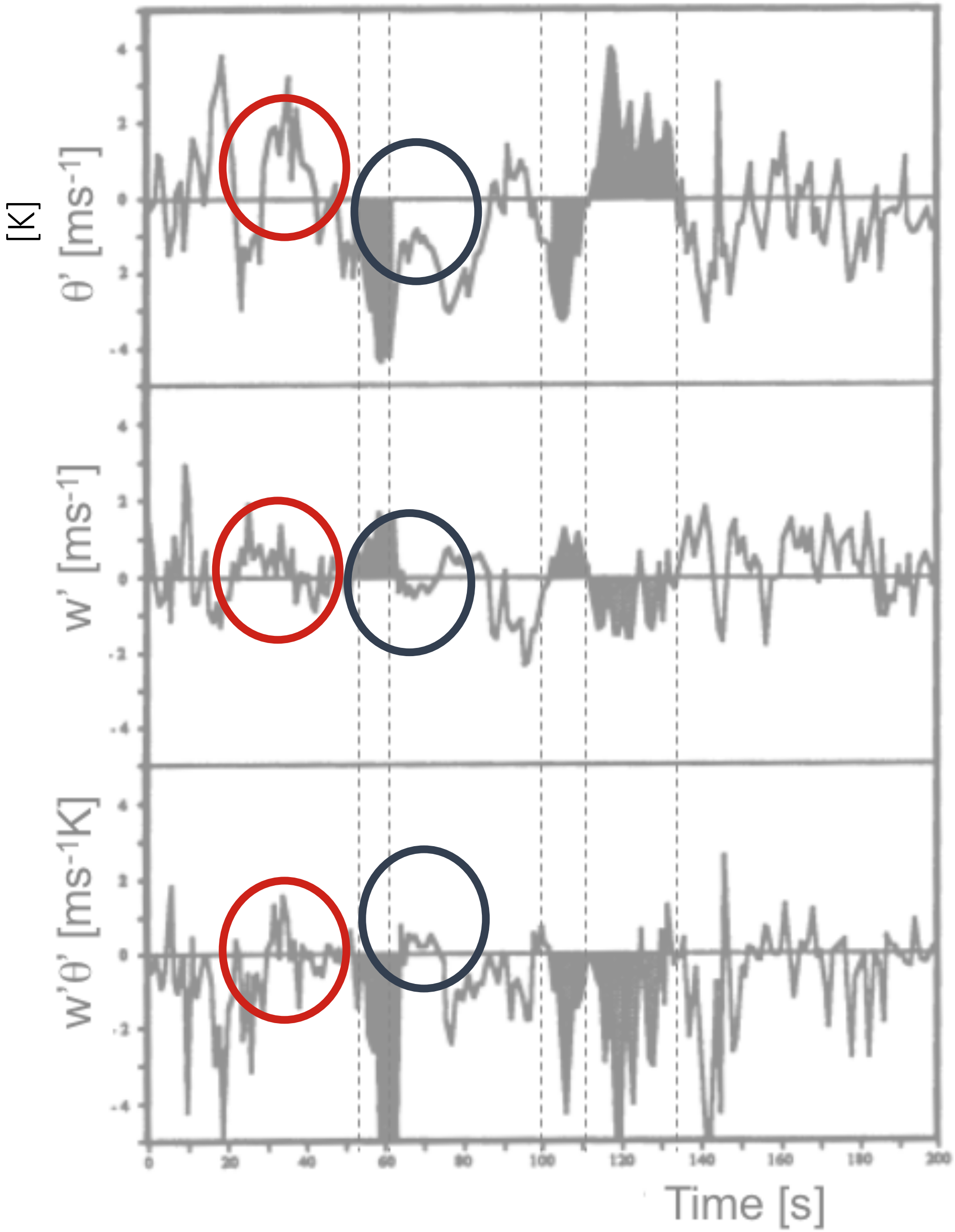


downward transport:

θ' > 0
w' < 0
} relatively warm air goes ↓

θ' < 0
w' > 0
} relatively cold air goes ↑

both:
downward transport of heat



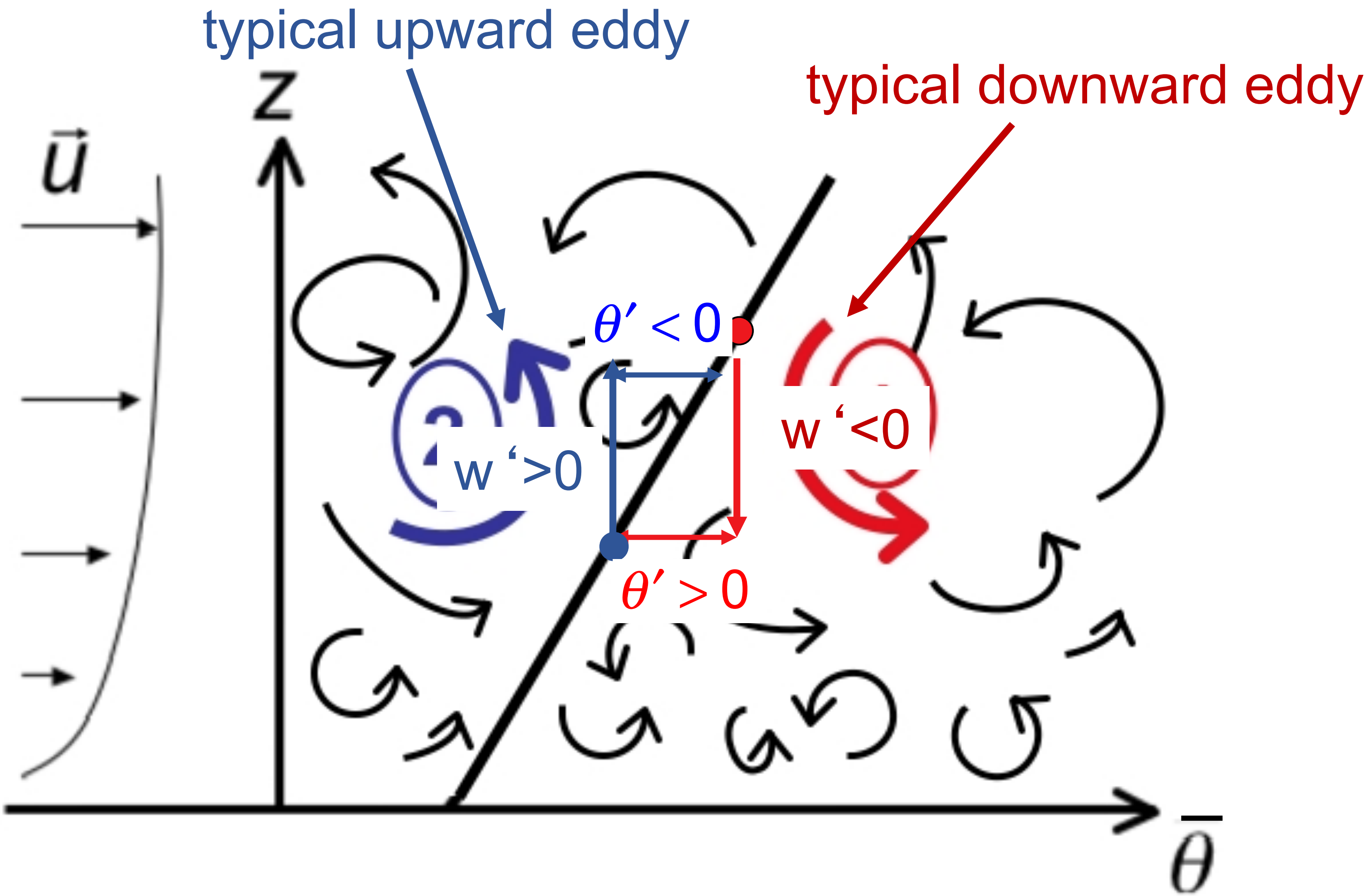
upward transport:

$\theta' > 0$, $w' > 0$ } relatively warm air goes ↑

$\theta' < 0$, $w' > 0$ } relatively cold air goes ↓ (→ transports heat deficit ↓)

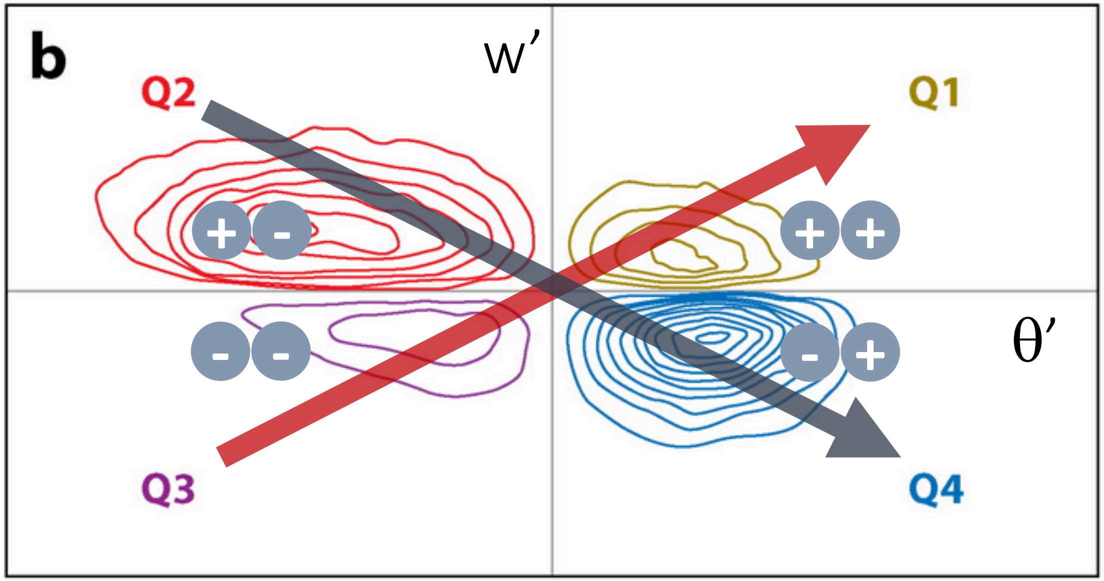
both:
upward transport of heat

Turbulent Fluxes



Quadrants

Upward kinematic heat flux



instantly:

$w'\theta' < 0$

$w'\theta' > 0$

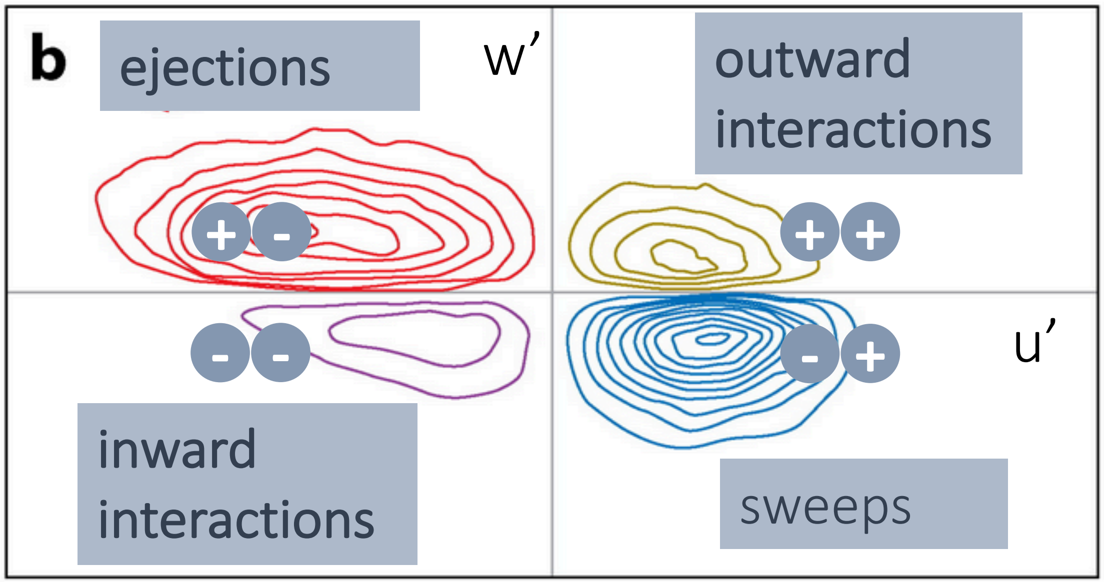
average:

$$\overline{w'\theta'} \neq 0$$

Downward kinematic heat flux

Quadrants

Upward kinematic momentum flux



Downward kinematic momentum flux

instantly:

w'u' < 0

w'u' > 0

average:

$$\overline{w'u'} \neq 0$$

→ investigate type of transport

Quadrant analysis: Momentum Transport

equilibrium conditions (*Surface Layer*):

total stress = ejections + sweeps + OutIn + InIn

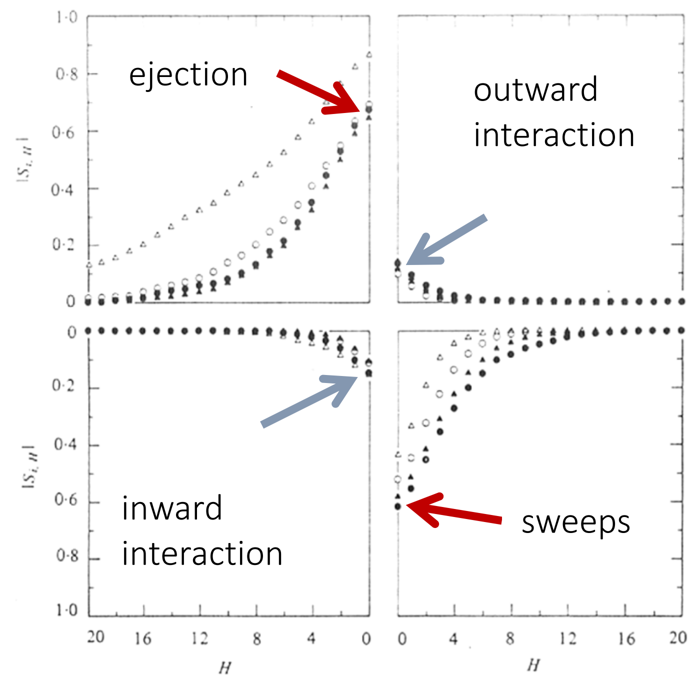
	ejections	sweeps	OutIn	InIn
	w'>0	w'<0	w'>0	w'<0
	u'<0	u'>0	u'>0	u'<0

$$\overline{u'w'}_{tot} = 0.6\overline{u'w'}_{tot} + 0.6\overline{u'w'}_{tot} - 0.1\overline{u'w'}_{tot} - 0.1\overline{u'w'}_{tot}$$

● ● ● ● homogeneous surface

△ △ △ △
○ ○ ○ ○ other surface types

H=0 include all eddy sizes



Raupach et al 1981

Quadrant analysis: Momentum Transport

urban canopy: