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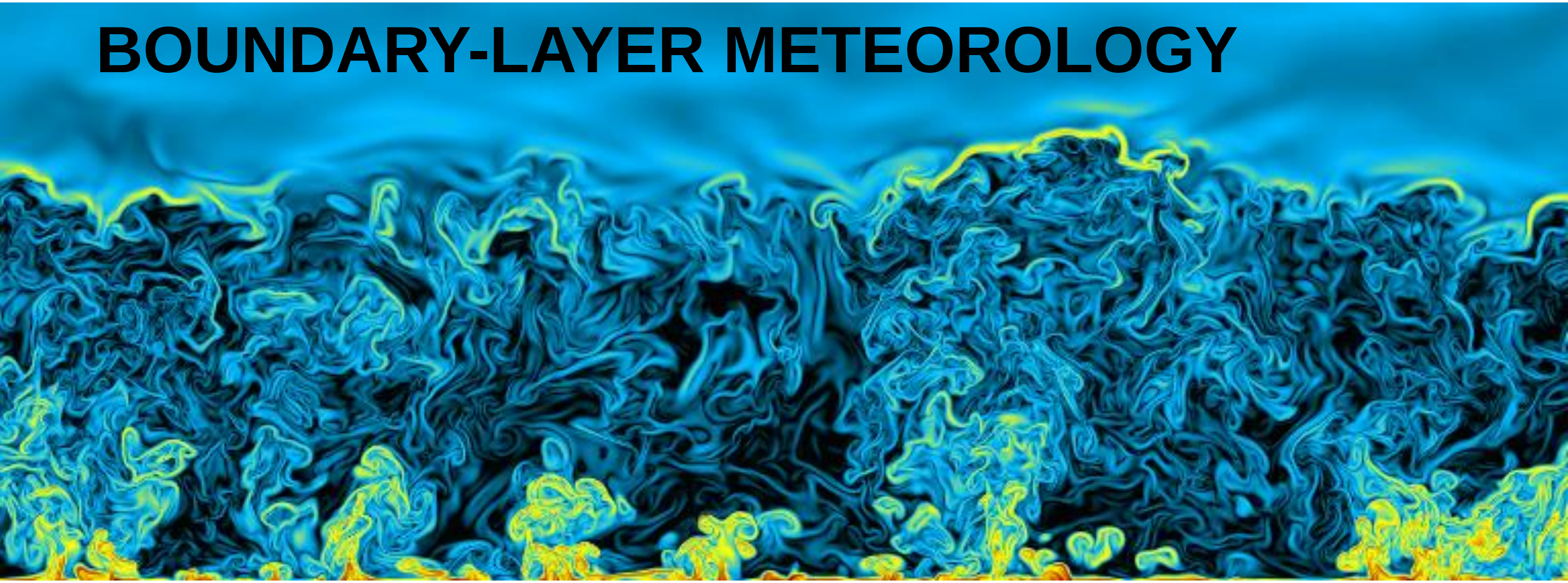
BOUNDARY-LAYER METEOROLOGY



Chapter 2: Turbulence

CONTENT

1. Introduction
2. A Brief Introduction to Atmospheric Turbulence
3. Statistical Treatment of Turbulence
4. Similarity Theory
5. Conservation Equations for Turbulent Flows
6. Turbulent Kinetic Energy and Dynamic Stability
7. Turbulence Spectra
8. Synthesis

CONTENT

2. A Brief introduction to Atmospheric Turbulence

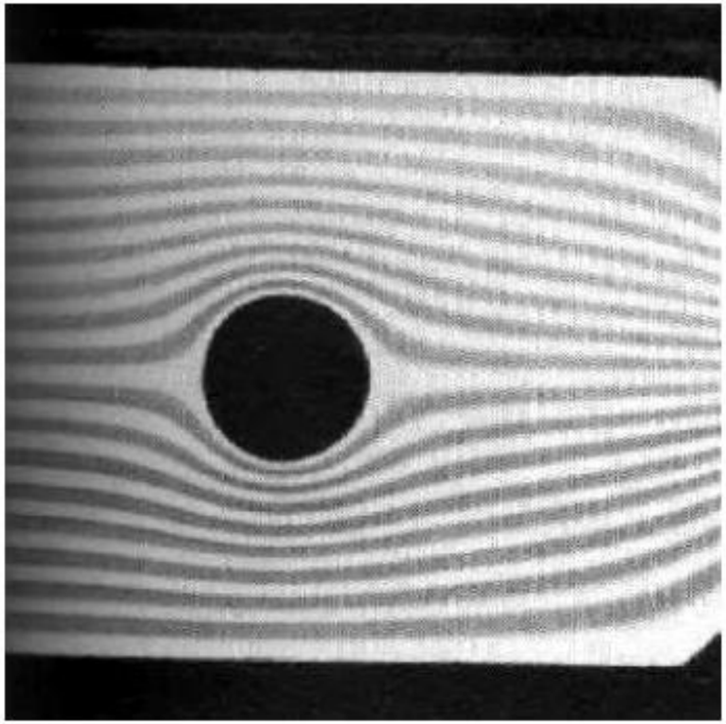
- What is turbulence?
- Laminar vs. turbulent flow
- The Reynolds number
- The turbulence syndrome
- Viscosity

TURBULENCE

- dominant property of the PBL
- opposite states: turbulent ↔ laminar
- flow characteristic of fluids

- working definitions:
  → laminar flow = orderly flow
  → turbulent flow = irregular, 'chaotic', individual

LAMINAR VS TURBULENT



(a) Laminar flow

LAMINAR VS TURBULENT

...in the atmosphere

TURBULENCE

- dominant property of the PBL
- opposite states: turbulent ↔ laminar
- flow characteristic of fluids
- working definitions:
  → laminar flow = orderly flow
  → turbulent flow = irregular, ‘chaotic’, individual

Measure: Reynolds number

OSBORNE REYNOLDS (1842-1912)

Osborne Reynolds at the age of ∼ 24 (ca. 1866).

OSBORNE REYNOLDS (1883)

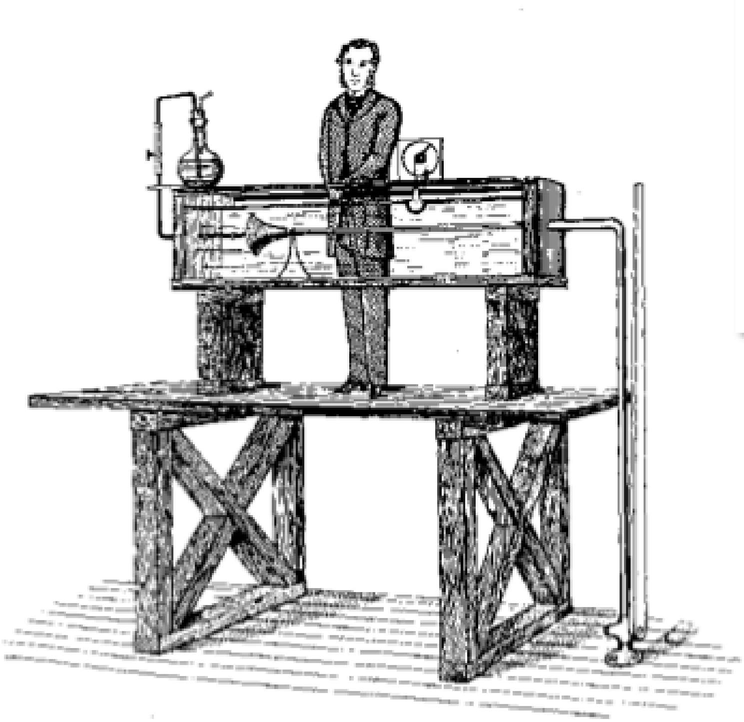


Figure 1 Artist's concept of Reynolds' flow-visualization experiment.

Boundary Layer Meteorology |Turbulence

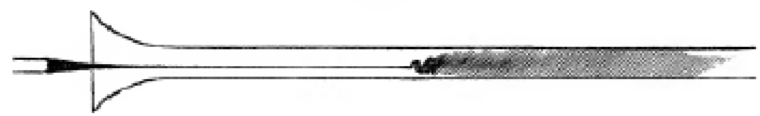
III. "An Experimental Investigation of the Circumstances which Determine whether the Motion of Water shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channels." By OSBORNE REYNOLDS, F.R.S. Received March 7, 1883.

(Abstract.)

1. *Objects and Results of the Investigation.*—The results of this investigation have both a practical and a philosophical aspect.

In their practical aspect they relate to the *laws of resistance to the motion of water in pipes*, which appears in a new form, the law for all velocities and all diameters being represented by an equation of two terms.

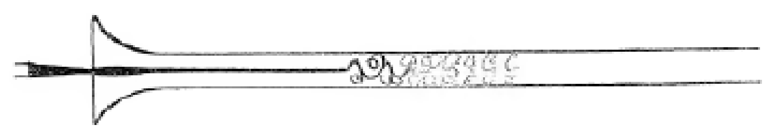
FIG. 4.



Any increase in the velocity caused the point of breakdown to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colours resolved itself into a mass of more or less distinct curls showing eddies, as in fig. 5.

FIG. 5.



REYNOLDS NUMBER

[Osborne Reynolds, 1883]

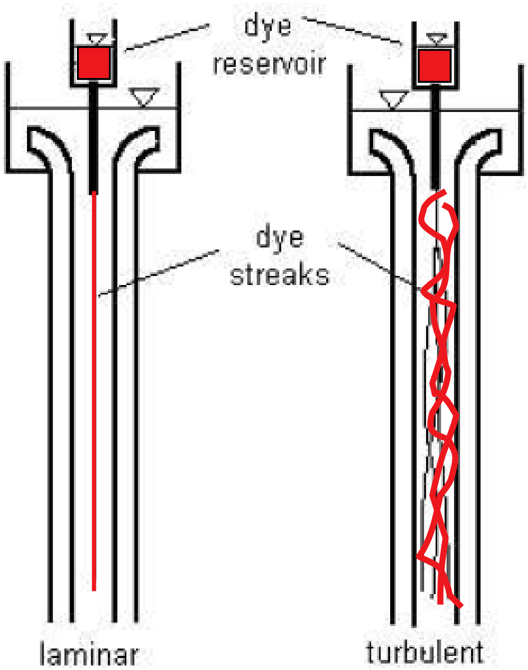
Experiments with flow in a tube

- if $u < u_{crit}$: laminar flow
- if $u > u_{crit}$: turbulent flow

- different tube, different fluid
  → general result

$$Re = \frac{U_* L_*}{\nu}$$

U_*	characteristic velocity
L_*	characteristic length
ν	molecular viscosity



OSBORNE REYNOLDS (1883)

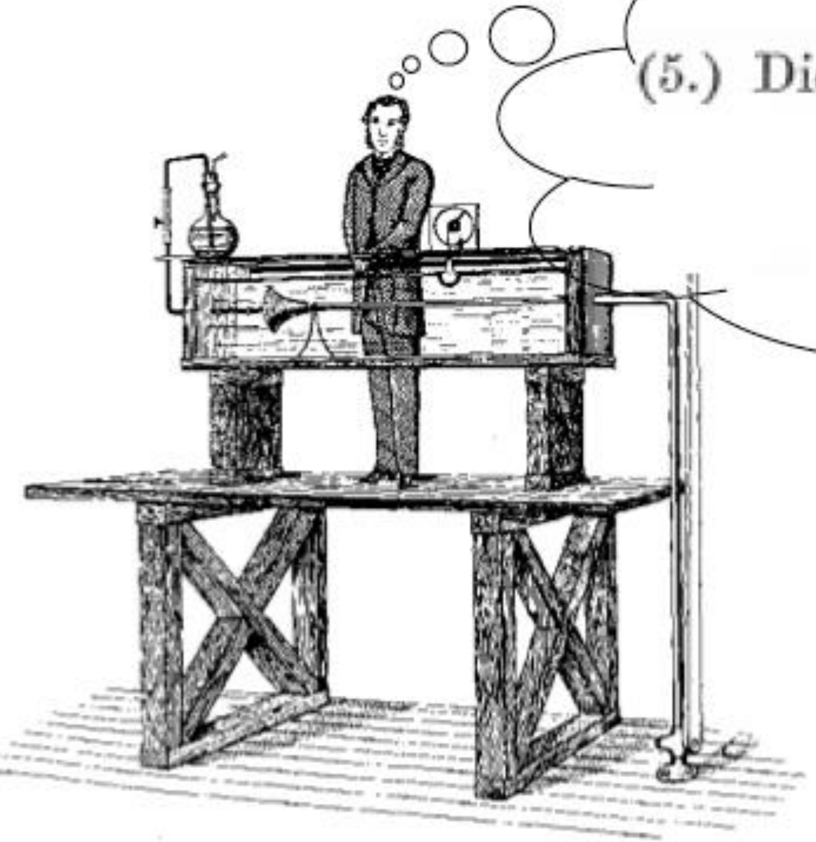


Figure 1 Artist's concept of Reynolds' flow-visualization experiment.

(5.) Did the eddies come in at a certain value of

$$\frac{\rho c U}{\mu}?$$

U : characteristic velocity

C : radius of the tube

ρ : density of air

μ : dynamic viscosity

REYNOLDS NUMBER

Re is a measure for turbulence

Re > 2000: turbulent
Re < 2000: laminar

} experience, approximation

Re describes ratio external/inner forces

drag force:
(external forces)

$$\left|\vec{F}_D\right| \approx u \frac{\partial u}{\partial x} \propto \frac{U_*^2}{L_*}$$

molecular friction:
(inner forces)

$$\left|\vec{F}_{mf}\right| \approx \nu \frac{\partial^2 u}{\partial x^2} \propto \nu \frac{U_*}{L_*^2}$$

REYNOLDS NUMBER

- $\left|\vec{F}_D\right|$ dominant: flow becomes 'unstable' (disturbances can grow) → turbulent

- $\left|\vec{F}_{mf}\right|$dominant: strong inner bondings (disturbances wiped out) → laminar

→ Def:

$$Re = \frac{\left|\vec{F}_D\right|}{\left|\vec{F}_{mf}\right|} = \frac{U_* L_*}{\nu}$$

- turbulence is a property especially of the flow (U_*, L_*) and of the fluid (ν)

Re FOR A WATER PIPE

$$Re = \frac{U_* L_*}{\nu}$$

viscosity of water: $\nu_W \approx 10^{-6}$ m^2 s^{-1}

length scale: $L_* = 10^{-2}$ m

velocity scale: $U_* < 0.2$ m s^{-1}

laminar for

$\Longrightarrow$ $Re < 2000$

Re IN THE PBL

$$Re = \frac{U_* L_*}{\nu}$$

viscosity of air: $\nu_a = 1.5\ 10^{-5}$ m^2 s^{-1}

length scale: $L_* =$1000 m

velocity scale: $U_* =$1 – 10 m s^{-1}

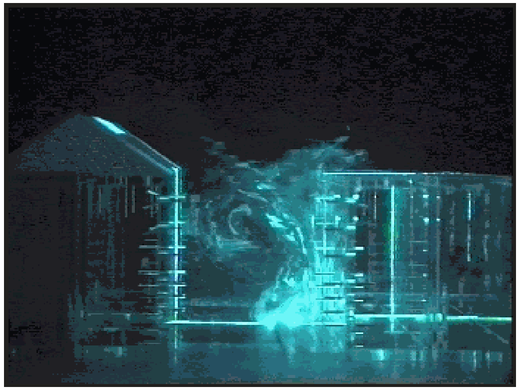
$$\Rightarrow Re_{PBL} = \frac{U_* L_*}{\nu} = \frac{2}{3} 10^8 \gg 2000$$

even for small-scale phenomena:

→ e.g., flow in a street canyon

→ U_* small, L_* small

→ Re >> 2000



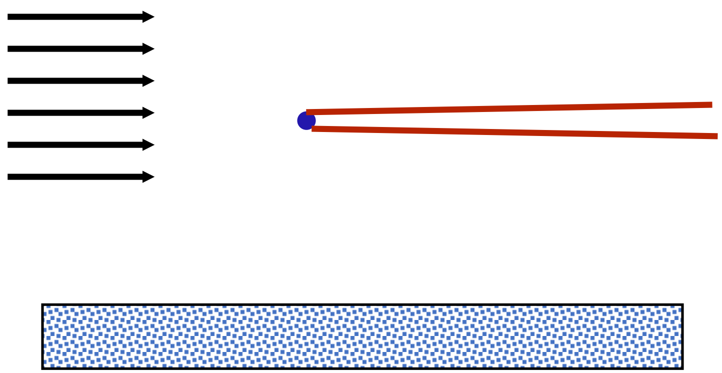
TURBULENCE IN THE PBL

The ABL is that part of the atmosphere that is dominated by turbulence

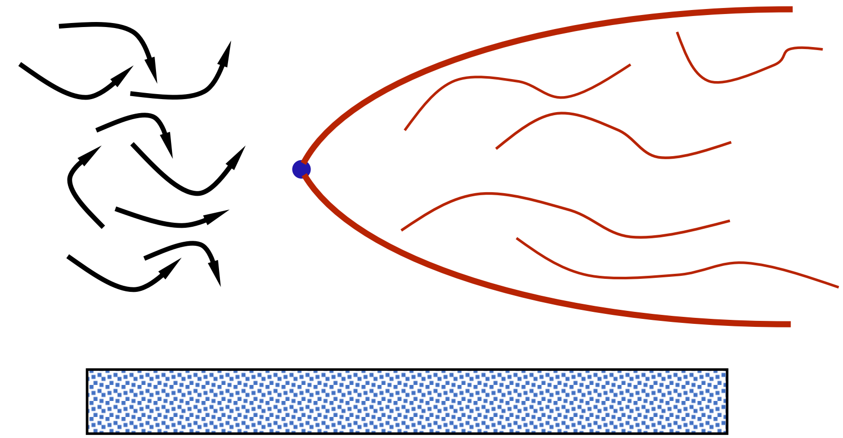
TURBULENCE IN THE PBL

Re > 2000?

MOLECULAR VS. TURBULENT DIFFUSION



Laminar: molecular diffusion



Turbulent: turbulent diffusion

10^6 times more effective!

TURBULENCE SYNDROME

Flow	Turbulence is a property of the flow not the fluid and hence no property of the material (still the latter plays a role in how easily a fluid comes in a turbulent state). Also, turbulence is not possible without any movement.
Irregularity	Turbulence is an example of deterministic chaos. This means that even if the flow is determined by deterministic conservation equations, the sensitive dependence on the initial conditions makes it impossible to ever predict its exact future state in every detail. Even if we have a model that resolves all the spatial scales of atmospheric turbulence, it only predicts *one possible future state* from the many possible
Mixing	Turbulent mixing is very efficient, roughly 10^6 times more effective than molecular diffusion. Hence it is almost exclusively responsible for exchange of momentum, heat and mass from and to the Earth's surface and throughout the ABL. The effect of this mixing property is, in general, a weakening of gradients.
Vorticity / Rotation	Turbulence is rotational, i.e. the rotation of the vector fields is in general non-zero. Therefore, an attractive way to look at turbulent flows is that of a superposition of eddies of different spatial scales. Turbulent motion is always three-dimensional. Even if some aspects of two-dimensional structures resemble turbulence in some respects they are not turbulent. A prominent example may be surface waves, which are chaotic and random, but never truly turbulent.

Ivana Stiperski (ivana.stiperski [at] uibk.ac.at)

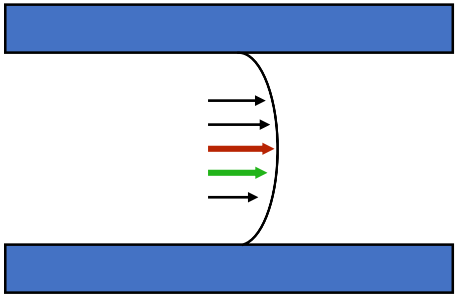
TURBULENCE SYNDROME

Dissipation	Without continuous energy input turbulence quickly decays through dissipation, i.e. the transformation of turbulent kinetic energy into heat. Waves are not dissipative and therefore not turbulent.
Continuum	In a fully turbulent flow, turbulent *eddies of all spatial scales* are present. There may be one or more preferred eddy sizes (due to the flow's configuration) but the spectral distribution does not exhibit complete gaps. Even the smallest scales of atmospheric turbulence ($\sim 10^{-3}$m) are much larger than the free path length for 'air' ($\sim 10^{-7}$m).
Configuration	The turbulence state of a flow is entirely dependent on its environment. Problems in turbulence are problems of the boundary conditions.
Non-linearity	Turbulent flows are highly non-linear. Processes, which contribute to turbulence, are always coupled through feedback mechanisms.

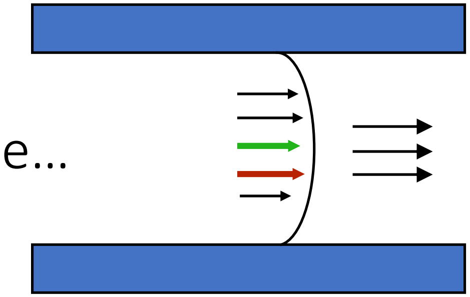
HOW DOES TURBULENCE EVOLVE?

- small irregularities exist in any flow
  → molecular: Brownian motion
  → density differences
  → boundaries
- if they can grow:
  → unstable
  → turbulent
- when can disturbances grow?
  → suitable conditions….
  i.e., large gradients (wind shear due to friction)
  i.e., stratification

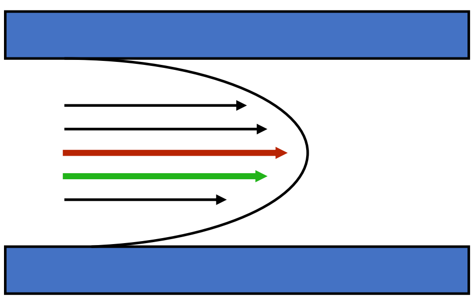
HOW DOES TURBULENCE EVOLVE?



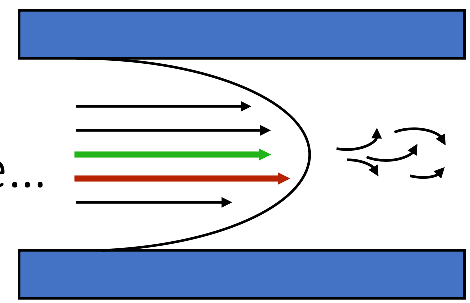
small disturbance…



→ small velocity
→ small gradients
→ relatively insensitive



small disturbance…



→ large velocity
→ large gradients
→ ‘unstable’

HOW DOES TURBULENCE EVOLVE?

Conditions for the 'production' of turbulence

- large gradients
- '*boundary* layer'
    - friction: gradient in flow speed → mechanical turbulence
    - exchange of heat at the surface: large density *gradient* of medium through which radiation propagates → thermally produced turbulence

VISCOSITY

A property of the fluid in Re

General derivation

→ first: Toolbox…

TOOLS

1) Coordinate system

- Cartesian, right handed: (x, y, z) or (x_1, x_2, x_3)
- Wind components: (u, v, w) or (u_1, u_2, u_3)
- u: longitudinal (flow) direction
  v: lateral direction
  w: vertical direction

TOOLS

2) Einstein Summation (convention)

a): $$\frac{\partial u_i}{\partial x_i} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}$$ Index *i* appears twice

b): If an index appears only once in a term, it is not a summation index for any term of the equation → 3 **equations**

$$\frac{\partial u_i}{\partial t} = F_{G,i} + F_{P,i}$$ only one *i* per term

→

$$\frac{\partial u_1}{\partial t} = F_{G,1} + F_{P,1}$$
$$\frac{\partial u_2}{\partial t} = F_{G,2} + F_{P,2}$$
$$\frac{\partial u_3}{\partial t} = F_{G,3} + F_{P,3}$$

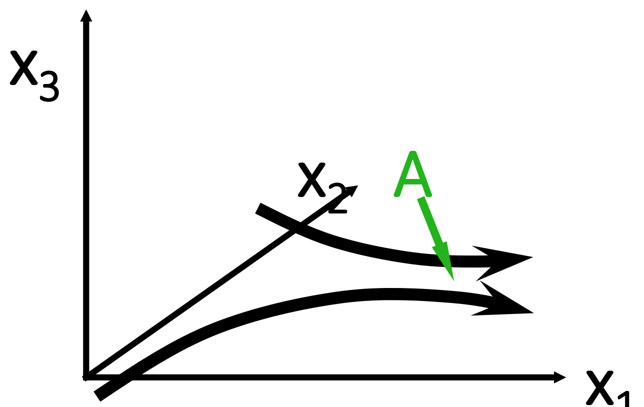
TOOLS

3) Incompressibility

→ $\frac{\partial u_i(\vec{x},t)}{\partial x_i} = 0$ (summed!)

→ special form of the mass conservation equation

→ atmospheric BL: ‘always and everywhere’ fulfilled



Three possibilities:

1. air exits towards the top
2. flow accelerates
3. density in A increases

1 & 2: incompressible flow

4) Newtonian fluid

Shear stress proportional to deformation rate

$$\sigma_{ij} = \rho\nu S_{ij} \equiv \rho\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

σ_{ij} shear stress tensor

S_{ij} deformation rate

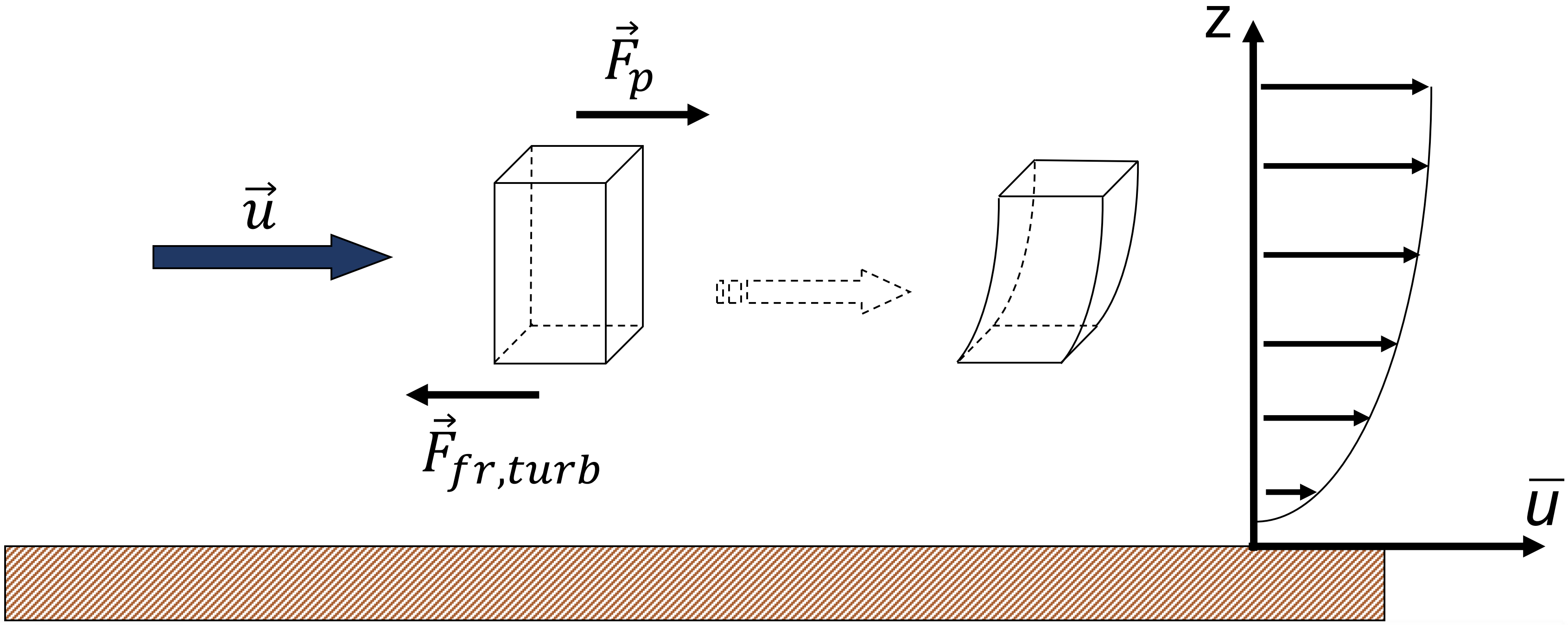
ν kinematic viscosity

$\rho\nu$ proportionality factor

4) Newtonian fluid

Shear stress proportional to deformation rate

$$\sigma_{ij} = \rho \nu S_{ij} \equiv \rho \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$



VISCOSITY

Consider the momentum conservation equation

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{x_j} = -\delta_{i3} g - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}$$

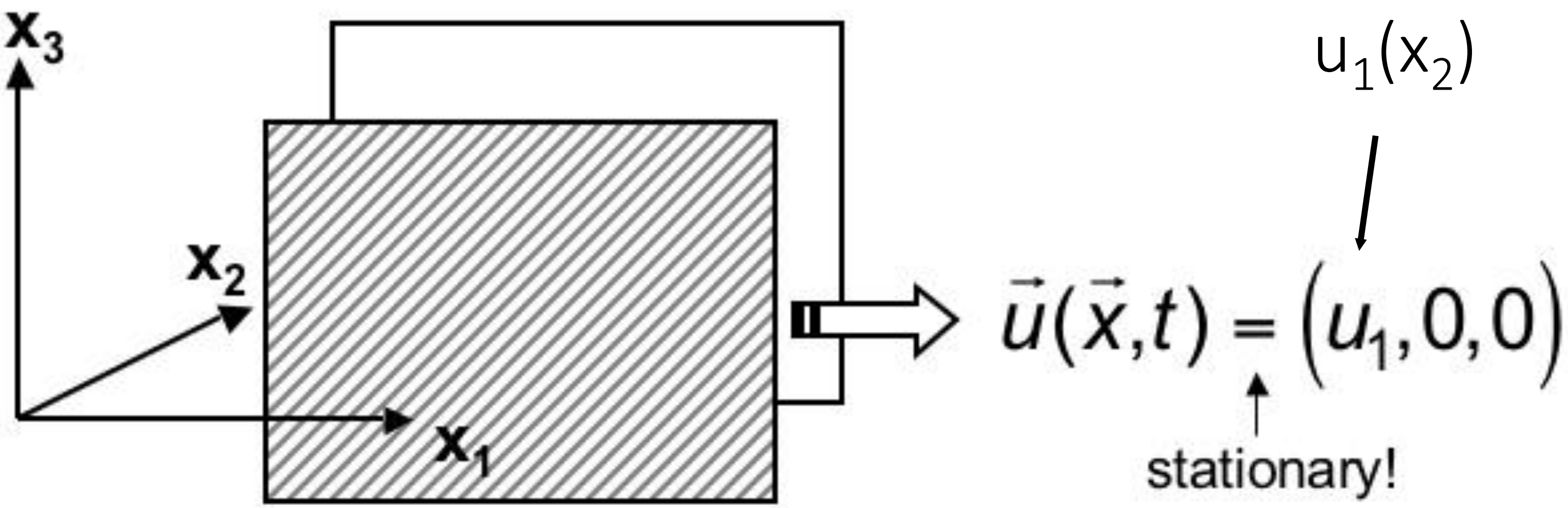
Simplest case:

→ no density differences (neutral)

→ stationary shear flow

→ between 2 parallel plates in xz- plane

VISCOSITY



$$\cancel{\frac{\partial u_i}{\partial t}} + \cancel{u_j \frac{\partial u_i}{x_j}} = \cancel{-\delta_{i3} g} - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}$$

$$i = 1: \quad u_1 \frac{\partial u_1}{\partial x_1} = 0 \qquad u_2 \frac{\partial u_1}{\partial x_2} = 0 \qquad u_3 \frac{\partial u_1}{\partial x_3} = 0$$

$$i = 2{,}3: \quad u_2 = u_3 = 0$$

VISCOSITY

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{x_j} = -\delta_{i3} g - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}$$

$$\frac{\partial p}{\partial x_1} = \frac{\partial \sigma_{1j}}{\partial x_j}$$

forcing in x_1 direction

Newtonian fluid: $\sigma_{ij} = \rho\nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$

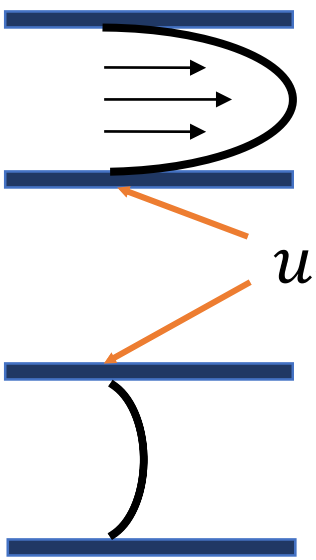
$$\frac{\partial p}{\partial x_1} = \frac{\partial \sigma_{12}}{\partial x_2} = \rho\nu \frac{\partial^2 u_1}{\partial x_2^2}$$

definition and simplest 'instruction' to determine dynamic viscosity

DYNAMIC VISCOSITY

$$\eta = \rho \nu$$

'Resistance', with which the fluid responds to external force with shear stress



$\frac{\partial p}{\partial x_1}$ large

$u_1 = 0$

$\frac{\partial p}{\partial x_1}$ small

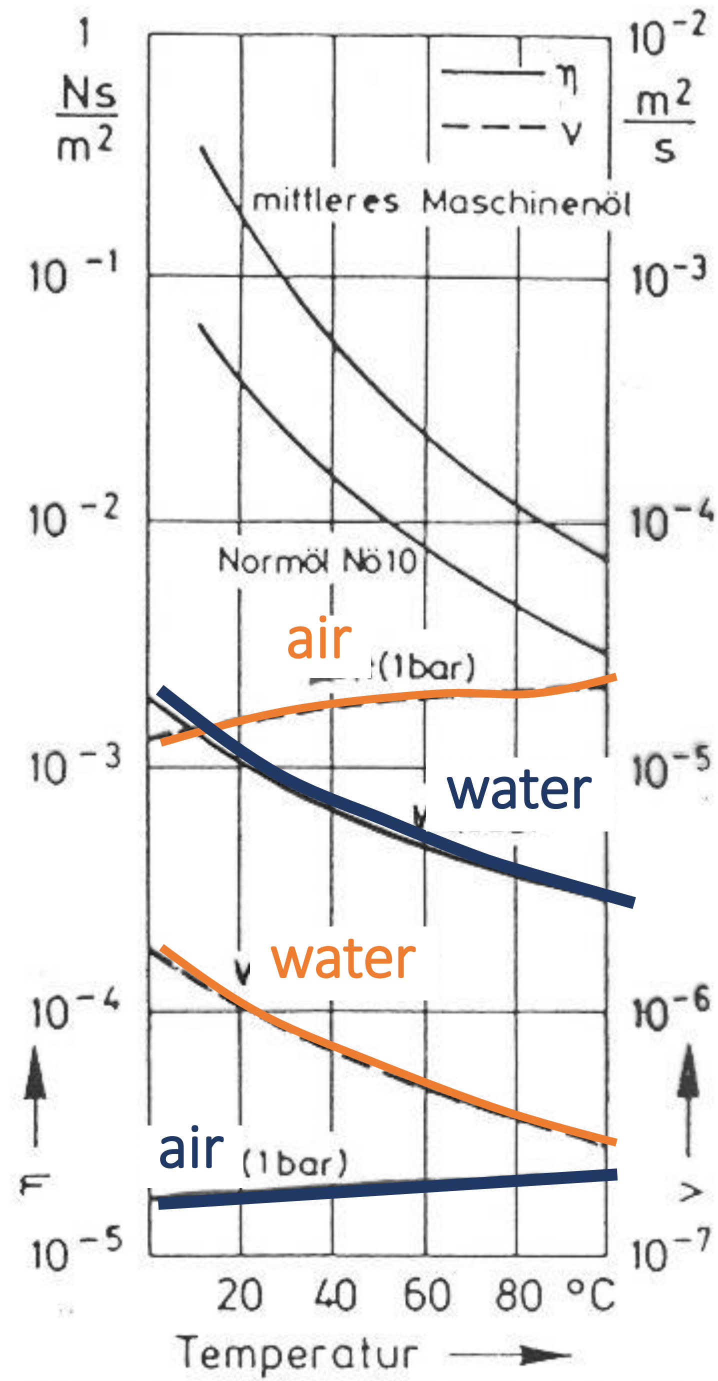
DYNAMIC VISCOSITY

$$\eta = \rho \nu$$

ν: air > water

η: water > air

- ν is the material constant
- η determines how the dynamic system responds to external forcing



VISCOSITY

From statistical mechanics: $\nu = \lambda\sqrt{\frac{k_B T}{m}}$

λ free path length [air: λ = 6.5×10^{-8} m]

k_B free Boltzmann constant [k_B =1.38×10^{-23} J K^{-1}]

T absolute temperature

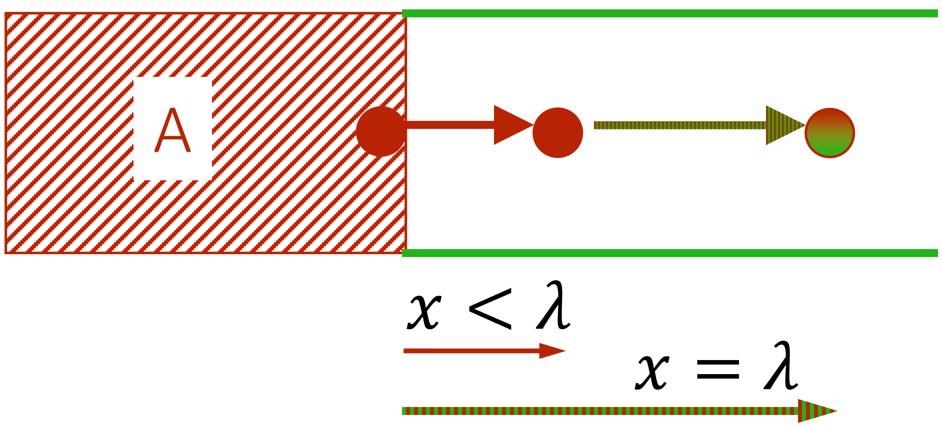
m particle mass [air: m_a = 5×10^{-26} kg]

→ ν_{air} = 1.5×10^{-5} m^2 s^{-1}

→ property of fluid, weakly T dependent

FREE PATH LENGTH

How far does a fluid element with property A travel into domain with property B before mixing becomes effective



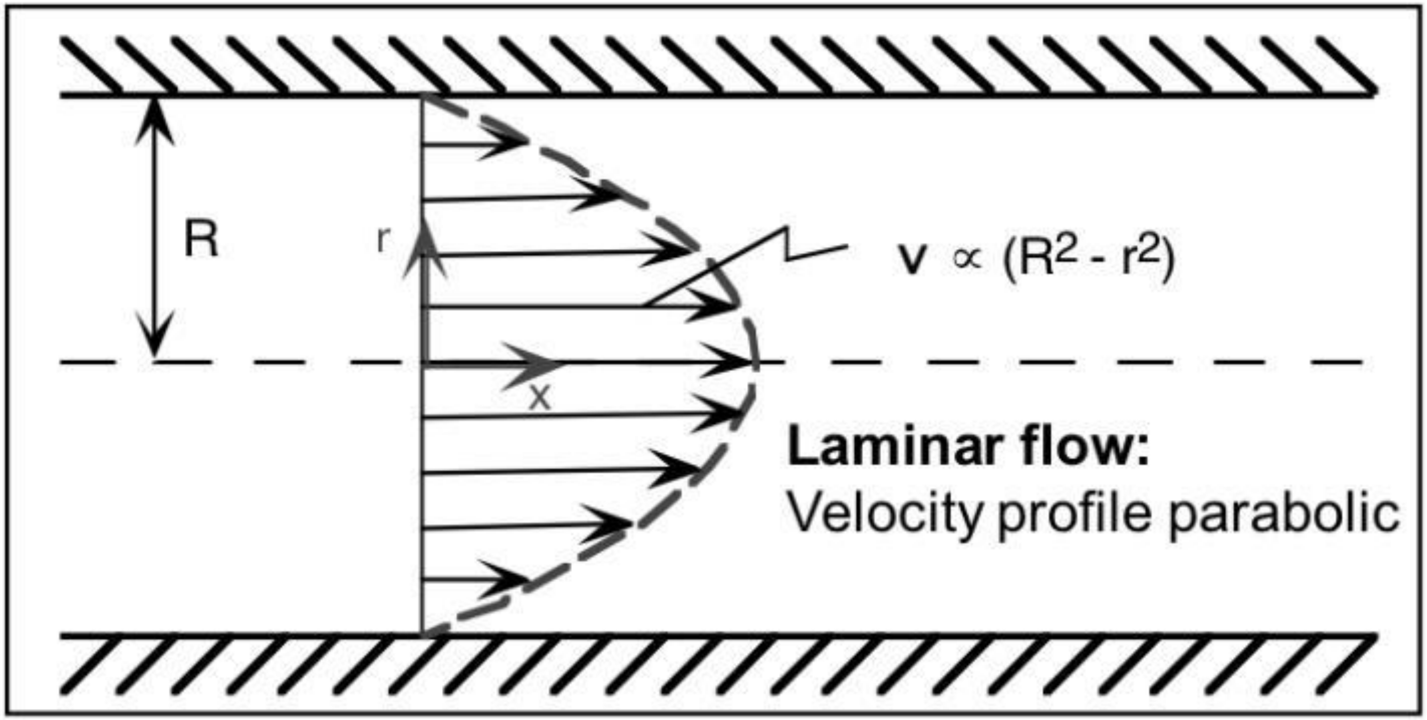
Motion:
→ Brownian
→ molecular diffusion

$$\nu = \lambda \sqrt{\frac{k_B T}{m}}$$

λ large ⇔ ν large [vice versa]

TURBULENT VS. LAMINAR

→ Channel flow: laminar



→ parabolic velocity profile

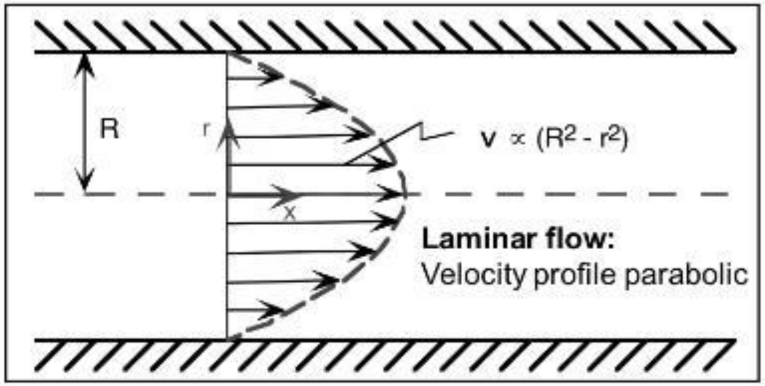
CHANNEL FLOW – PARABOLIC PROFILE

From above, simplest case: $\underbrace{\frac{\partial p}{\partial x_1}}_{f_1(x_1)} = \underbrace{\rho\nu \frac{\partial^2 u_1}{\partial x_2^2}}_{f_2(x_2)}$

only possible if $f_1(x_1) = f_2(x_2) = const$

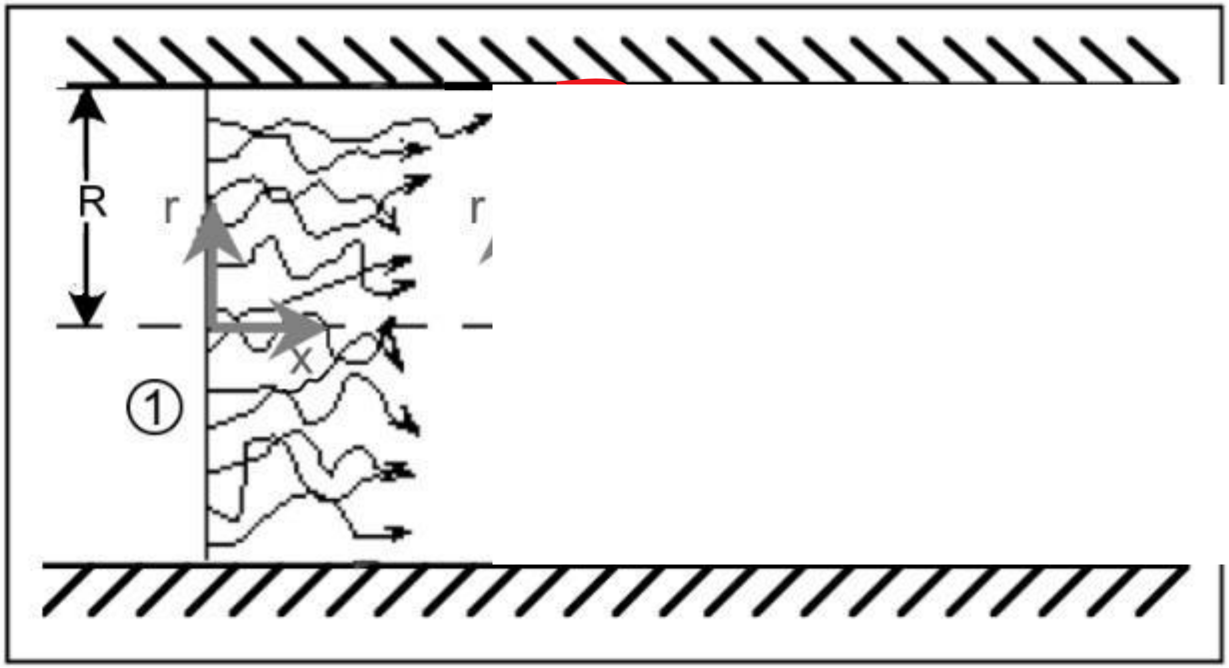
$$\rho\nu \frac{\partial^2 u_1}{\partial x_2^2} = const \longrightarrow u_1(x_2) \propto x_2^2$$

‘no slip condition’ @ surface:



TURBULENT VS. LAMINAR

→ Channel flow: turbulent



- trajectories cross
- instantaneous picture not equal to mean
- flat velocity profile

TURBULENT VS. LAMINAR

Difference:

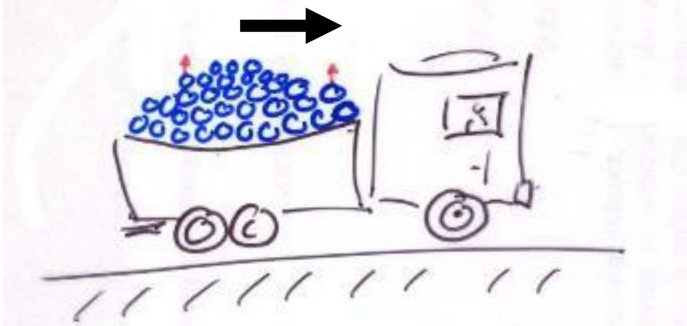
- ratio of forces
- up to now: influence of shear stress (inner forces)
- turbulent flow: external force, which makes the flow more unstable

TURBULENT VS. LAMINAR

Analogy

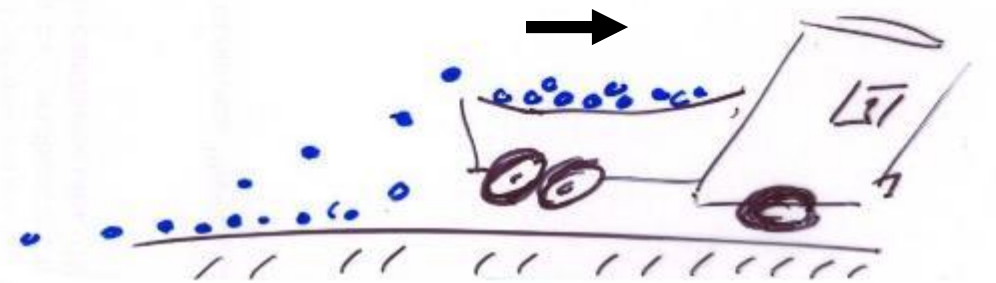
'laminar'

5 km/h



'turbulent'

150 km/h



Mixing Length

→ small mixing length
→ little mixing
→ molecular diffusion
→ viscosity (material)

→ large mixing length
→ strong mixing
→ turbulent diffusion
→ 'eddy viscosity' (flow)

TURBULENT VS. LAMINAR

Laminar flow

- mixing length proportional to the free path length
- viscosity

Turbulent flow

- mixing length proportional to R (domain)
- viscosity → eddy viscosity...

EDDY VISCOSITY

Newtonian fluid: $\sigma_{ij} = \rho\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

shear stress

dynamic viscosity
as proportionality factor

deformation rate

Turbulent flow (analogy): $\tau_{ij} = K_{ij}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)$

'Reynolds stress'
(turbulent shear stress)

mean
deformation rate

'Eddy Viscosity'
= 'turbulent viscosity'

TURBULENT VS. LAMINAR

	Laminar	Turbulent
Mixing length	~ **Free path length** [$\mathcal{O}(10^{-8}$ m)] Dependent on density and temperature → statistical state of fluid	~ **Dimension of domain** (e.g. R of pipe, distance from surface in the atmosphere) → Dependent on *flow*
Irregularities (without external forces)	**Microscopic** [L ~ λ] Balanced in ~ 10^{-9} s	**Macroscopic** (L ~ R) May 'survive' L/u=t, e.g. L = 0.1 m u = 1 ms^{-1} t= 0.1 s L = 1000 m u = 5 ms^{-1} t=200 s
Efficiency of exchange	**Molecular viscosity** [$\mathcal{O}(\sim 10^{-5} m^2 s^{-1})$] Dependent on microscopic kinetics (λ, T) → *property of the fluid*	**Eddy-Viscosity** ~ 10^6 ν Dependent on *macroscopic flow properties* → *not a property of the fluid*
Reynolds Number	**<2000**	**» 2000**

TURBULENCE IN THE ATMOSPHERE

- **Especially within PBL**
- Cu clouds
- ‘clear air turbulence’

- Within the PBL:
  → conditions favorable to instabilities
  → friction! → mechanical turbulence
  → change of density at the ground (heating / cooling)