2 A brief Introduction to Atmospheric Turbulence

2.1 The Turbulence Syndrome

Turbulence as such is difficult to thoroughly define due to the fact that a large number of different classes of *turbulent flows* exist. With this statement, however, we have already made an important discovery: *turbulence is essentially a property of the flow and not of the fluid*. Even honey may flow in a turbulent manner if we choose the flow conditions accordingly. But what are these flow conditions? For very simple flows, such as pipe flow, the answer can easily be given (see Section 2.2). For environmental flows, on the other hand, with a large number of degrees of freedom varying in many aspects, no universal definition of turbulence can be given.

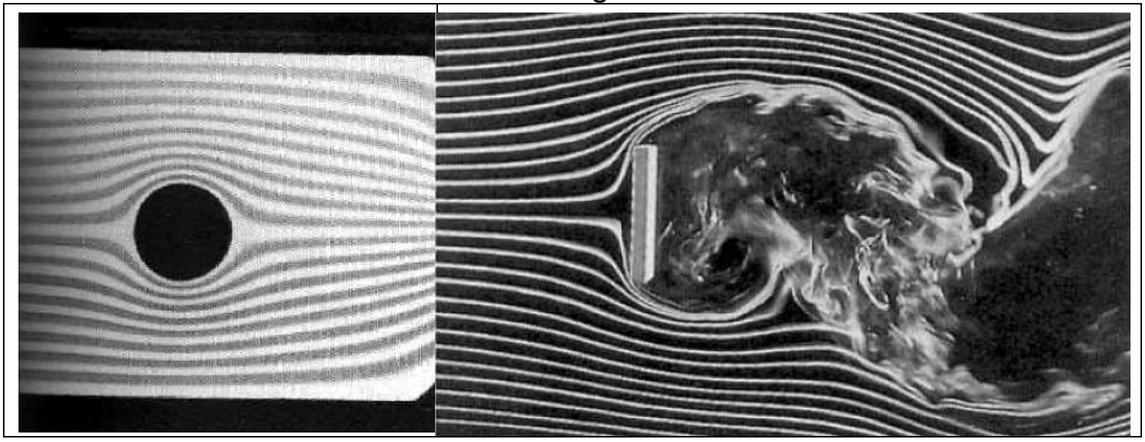

Figure 2.1 Examples of turbulent (right) and laminar (left) flows across an obstacle. From http://www.pi1.physik.uni-stuttgart.de/Vorlesungsversuche.

Let us start by looking at the opposite of turbulence. A *laminar* flow (Fig. 2.1) is an *orderly* flow (Mc Comb, 1992), in which the streamlines usually do not cross. Its properties are steady in time in the sense that a 'snap-shot' of one of its properties (e.g., a velocity profile) does not change from one instant to another. Also a laminar flow may evolve according to external influences (e.g., one or more forces acting on the fluid that change with time), but this change is slow as compared to the time it takes to establish the present state. As a working hypothesis, we may then view a turbulent flow as one with *dominating disorder*. If we can mark some fluid elements, e.g. by inserting some coloured dye into a fluid, successive snap shots of this flow will reveal an ever changing arrangement of fluid elements with respect to each other, and if we track individual fluid elements their streamlines may even have crossed after a while.

At first sight a turbulent flow is one that is not laminar. But even if it is difficult to devise a complete definition of turbulence a number of conditions can be given that must be present in a turbulent flow. We call the bulk of these conditions the *turbulence syndrome*. Its ingredients are summarised in Table 2.1.

The mixing property of turbulent flows is certainly the most influential for the ABL. Without this the near-surface temperature amplitude during the daily cycle would be strongly enhanced (less efficient transport of heat from and to the surface), water would not be available in the atmosphere to the degree it presently is and allows for precipitation to support agriculture and man's life (less efficient transport of water vapour into the atmosphere). Also, pollutants emitted from, say, an exhaust pipe would remain at high (deadly) concentrations for much longer times and distances from the source, thus making human life quite dangerous. The 'flow' and 'dissipation' properties describe some of the prerequisites to turbulent flow and the further entries address some of the difficulties we face when describing turbulent flows.

One of the pioneers of turbulence research, the famous British hydrodynamicist, Sir Horace Lamb has summarised those difficulties in 1932 when facing the end of his life (Goldstein 1969):

> *"...when I die and go to Heaven, there are two matters on which I hope for enlightenment. One is Quantum Electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really optimistic."*

2.2 The Reynolds Number

In the late 19th century Osborne Reynolds made a series of experiments in order to investigate the transition from laminar to turbulent flows. The hypothesis was to show that this transition is dependent on the ratio between the external forces acting on the fluid and the internal forces. The experiments were performed by studying the flow of different fluids at varying velocities and in pipes of different radii. Essentially, the external forces tend to make the flow unstable (prone to disorder) and the inner forces describe the fluid's viscosity. The external forces can be characterised through their over-all effect, i.e. the advection term in the conservation equation for momentum[1]

$$\text{External forces: } u\frac{\partial u}{\partial x} \propto \frac{U^2}{L} \tag{2.1}$$

Here, only the longitudinal velocity component is considered and an *order of magnitude* of the external forces is approximated by introducing a velocity scale (i.e. a typical or characteristic velocity) U and a length scale L.

The inner forces, on the other hand are characterised by the viscosity term in the conservation equation, viz.

$$\text{Internal forces: } \upsilon\frac{\partial^2 u}{\partial x^2} \propto \upsilon\frac{U}{L^2}, \tag{2.2}$$

where υ is the kinematic viscosity, a measurable property of the fluid. The ratio of the two has become known as the *Reynolds Number*

$$\mathrm{Re} =: \frac{UL}{\upsilon}\left(=\frac{U^2/L}{\upsilon U/L^2}\right). \tag{2.3}$$

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[1] We come back to this equation in more detail later in this section

The experiments of Reynolds and many more since have indicated that…

…a flow is turbulent if Re *is larger than about 2000*[2]*.*

We first of all note that the velocity (U) and the domain (L) determine whether or not a flow is turbulent and second that nevertheless a property of the fluid, namely its viscosity, has an influence too.

Tab. 2.1: The Turbulence-Syndrome: Its most important ingredients

Flow	Turbulence is a property of the flow not the fluid and hence no property of the material (still the latter plays a role in how easily a fluid reahes a turbulent state). Also, turbulence is not possible without any movement.
Irregularity	Turbulence is an example of deterministic chaos. This means that even if the flow is determined by deterministic conservation equations, the sensitive dependence on the initial conditions makes it impossible to ever predict its exact future state in every detail. Even if we have a model that resolves all the spatial scales of atmospheric turbulence, it only predicts *one possible future state* from the many possible
Mixing	Turbulent mixing is very efficient, roughly 10^6 times more effective than molecular diffusion. Hence it is almost exclusively responsible for exchange of momentum, heat and mass from and to the Earth's surface and throughout the ABL. The effect of this mixing property is, in general, a weakening of gradients.
Vorticity / Rotation	Turbulence is rotational, i.e. the rotation of the vector fields is non-zero in general. Therefore, an attractive way to look at turbulent flows is that of a superposition of eddies of different spatial scales. Turbulent motion is always three-dimensional. Even if some aspects of two-dimensional structures resemble turbulence in some respects they are not turbulent. A prominent example may be surface waves, which are chaotic and random to some degree, but never truly turbulent.
Dissipation	Without continuous energy input turbulence quickly decays through dissipation, i.e. the transformation of turbulent kinetic energy into heat. Waves are not dissipative and therefore not turbulent.
Continuum	In a fully turbulent flow, turbulent *eddies of all spatial scales* are present. There may be one or more preferred eddy sizes (due to the flow's configuration) but the spectral distribution does not exhibit complete gaps. Even the smallest scales of atmospheric turbulence ($\sim 10^{-3}$m) are much larger than the free path length for 'air' ($\sim 10^{-7}$m).
Configuration	The turbulence state of a flow is entirely dependent on its environment. Problems in turbulence are problems of the boundary conditions.
Non-linearity	Turbulent flows are highly non-linear. Processes, which contribute to turbulence, are always coupled through feedback mechanisms.

Often, it is not easy to choose the relevant length and velocity scales. Clearly for a pipe flow (as in Reynolds' experiments), the diameter of the pipe and the maximum in the parabolic velocity profile (see below, Fig. 2.3) are appropriate choices. But how is it for a channel flow? Is the length scale the channel's

[2] Clearly, using careful experimental technique this threshold can be pushed to either side: for example a laminar flow can be kept laminar by increasing the velocity up to values, which yield $\mathrm{Re} > 2000$.

width or its depth? Or even its length? Usually, the relevant scale is a limiting scale – for example in a channel flow either the width *or* the depth can become the relevant length scale.

Example:

Let us take as an example the flow through an ordinary tap water pipe with a diameter of L=0.01m and the kinematic viscosity of water being $\upsilon_w \approx 10^{-6} m^2 s^{-1}$. Then this flow is expected to be laminar for a velocity smaller than about 0.2 ms^{-1} and turbulent for higher velocities.

Now, in the atmospheric boundary layer, the kinematic viscosity of air at environmental temperatures is roughly $\upsilon_a = 1.5 \cdot 10^{-5} m^2 s^{-1}$. Hence, from the largest possible length scales (a few thousands of meters, i.e. the daytime depth of the CBL) down to, say, the typical depth of the surface layer (100m) and at any reasonable wind speed we obtain a Reynolds number many orders of magnitude larger than 2000. This is why we can usually state that the ABL is that *part of the atmosphere, which is dominated by turbulence*. Only at very low (essentially vanishing) wind speeds and very close to the surface the flow becomes laminar. Indeed, over regular surfaces the lowest few millimetres where the Reynolds Number drops below 2000 are called *Laminar Sublayer*.

2.3 Laminar vs. Turbulent Flows

To understand the difference between laminar and turbulent flows we may have a closer look at a pipe flow (as in Reynolds' experiments, see Fig. 2.2). Whatever the force is that drives the flow, its velocity will vanish at the surface. This is called the *no-slip condition*. If the force is large, then *U*, the maximum velocity in the centre of the pipe is also large, and the parabola describing the flow profile[3] will be steep. Small disturbances in the flow are always present due to, e.g., small density fluctuations. Such a small disturbance can easily lead to fluid elements finding themselves in an environment with different velocity, thus giving way to even larger (local) velocity gradients. Hence a small disturbance can grow and this finally leads to a flow with large disorder, i.e. a turbulent flow.

2.3.1 Viscosity

Although it has been stated that turbulence is a property of the flow and not the fluid, the viscosity plays an important role in defining whether or not a flow is turbulent. To understand its meaning we consider the momentum conservation equation for a simple incompressible, non-buoyant (i.e. neutral) flow[4]

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\delta_{i3} g - \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial \sigma_{ij}}{\partial x_j}, \qquad (2.4)$$

[3] Below, reasoning will be given, why this profile is parabolic.

[4] For the notation and some assumptions see the box 'Tools to describe atmospheric flows'

where g is the acceleration due to gravity, p is the atmospheric pressure and σ_{ij} the shear stress tensor.

For the present purpose a simple configuration of a stationary horizontal flow of a Newtonian fluid[4] between two infinite plates (Fig. 2.3) is considered. For

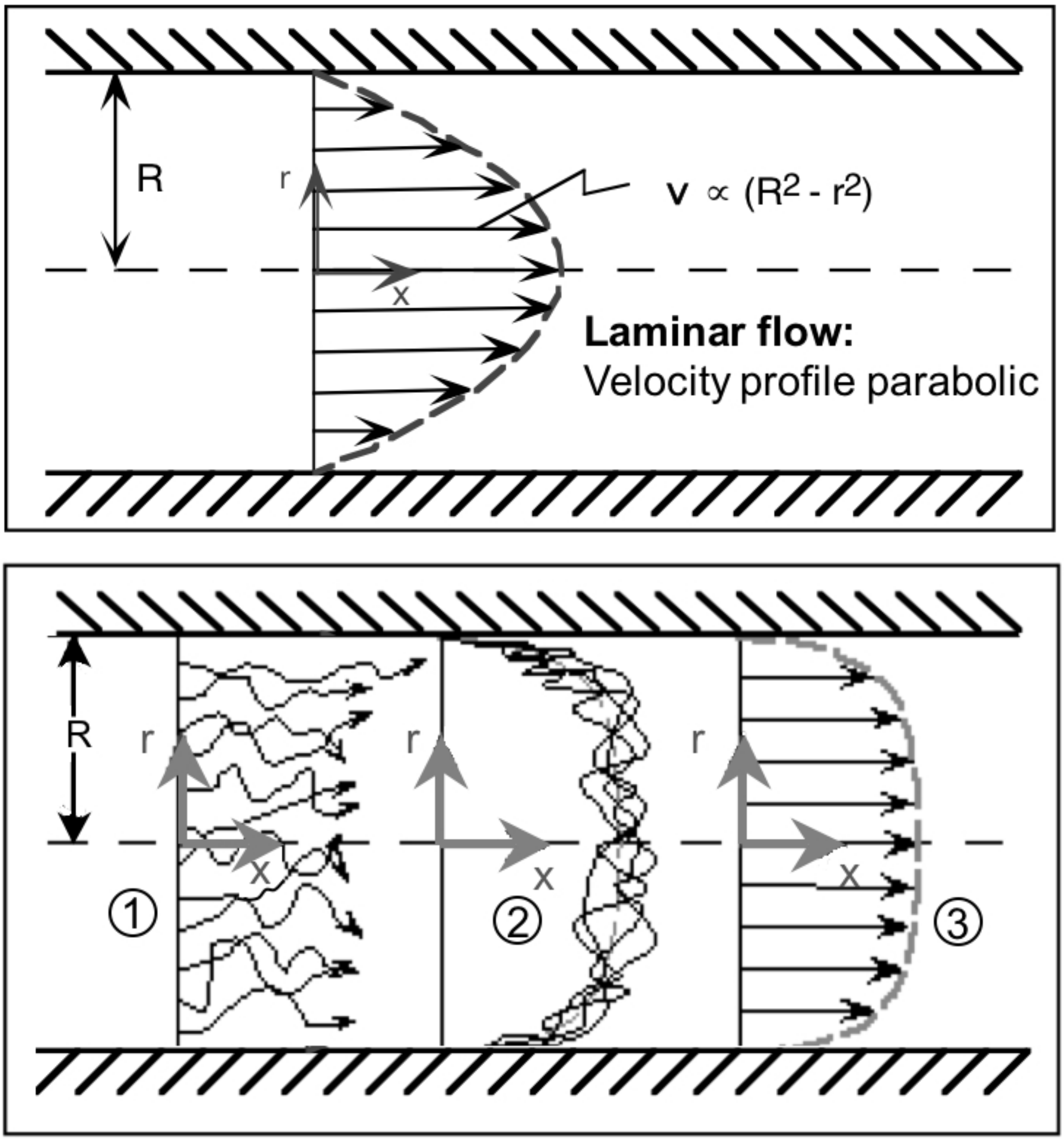

Figure 2.2 Laminar (upper panel) and turbulent (lower panel) flow in a pipe. In the laminar case the shown parabolic velocity profile is the *average* profile, but also true at every instant. In the turbulent flow the trajectories change every instant (1) and the instantaneous velocity profile changes accordingly (2). The resulting average velocity profile (3) is flatter than the parabolic of the laminar flow.

this situation $\vec{u}(\vec{x},t) = \left[u_1(x_2),0,0\right]$ and the pressure gradient must vanish in x_2 and x_3 directions, respectively. Hence, in the horizontal Eq. (2.4) reduces to

$$\frac{\partial p}{\partial x_1} = \rho \nu \frac{\partial^2 u_1}{\partial x_2^2} . \tag{2.5}$$

Equation (2.5) is definition and at the same time ‘a suitable plan’ to measure the kinematic viscosity, ν. From (2.5) we see that the proportionality factor between the forcing $\partial p / \partial x_1$ and the reaction in the fluid is

$$\eta = \rho \nu \tag{2.6}$$

and this is called the *dynamic viscosity*. Note that, for example, the kinematic viscosity of air is larger than that of water and for the dynamic viscosity, the opposite is the case, thus reflecting the ‘apparent’ experience with those two fluids.

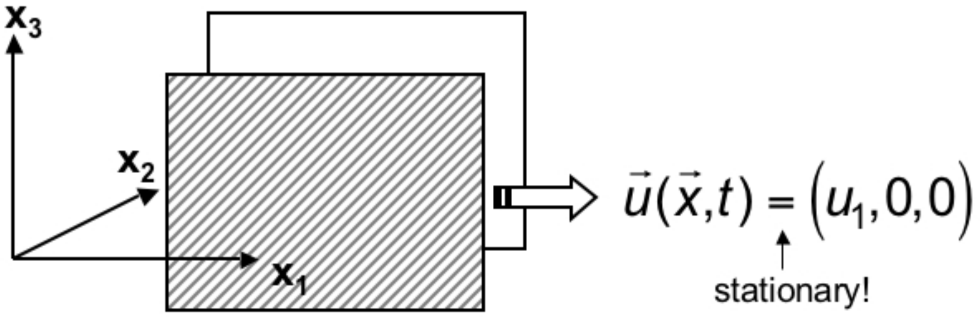

Figure 2.3 Simple flow configuration between two parallel plates.

The kinematic viscosity, being a property of the fluid, can also be determined using principles of statistical mechanics (Huang, 1987). It is beyond the scope of the present book to go into details, and therefore only the result is given here:

$$\nu = \lambda \sqrt{\frac{kT}{m}} \tag{2.7}$$

where λ is the free path length of the fluid, m the mass of a fluid element, k the Boltzman constant and T the absolute temperature.

The free path length can be interpreted as the distance a fluid element with property A can (statistically) travel within the fluid with property B until it has mixed through the action of *molecular diffusion.* A large λ corresponds to a large such distance (and vice versa) and hence reflects a large viscosity (example: try to stir a drop of jam into a spoon of honey and compare the external force – the stirring – required to that of mixing, say, water vapour from a hot cup into the ambient air).

2.3.2 Comparing laminar and turbulent flows

In Fig. 2.2 the velocity profiles through a pipe were compared for turbulent and laminar flows. In the case of the laminar flow, we may now use Eq. (2.5) to see that the left hand side (lhs) is a function of x_1 only, while the right hand side (rhs) is a function of x_2 alone. Hence they can only be equal in general if they are constant. And the rhs can be integrated twice to yield the parabolic profile (keeping in mind that the velocity must vanish at the surface due to the no-slip condition). For the turbulent flow on the other hand, the chaotic character of the flow leads to much more efficient exchange within the fluid elements. The instantaneous flow profile will change every instant. Still, on average a *characteristic* velocity profile is established that exhibits little variation in the centre of the pipe and shows steep gradients close to the wall. Hence, the result of the action of turbulence is an almost *well-mixed* average velocity profile (except close to the boundaries). We may thus state that the 'mixing length' (in analogy to the free path length of the laminar flow) is on the order of the pipe diameter. Later we will define an *eddy viscosity* - again in analogy to the definition of the molecular viscosity - relating the turbulent shear stress (called *Reynolds stress*) to the deformation of the mean flow. The properties of laminar and turbulent flows are summarised in Tab. 2.2

2.3.3 Turbulent flow in the Atmospheric Boundary Layer

The discussion of the Reynolds number has shown that atmospheric boundary layer flows are essentially always turbulent. In the previous paragraph we have seen that it is the no-slip condition near the boundary, which introduces the steep velocity gradients and this, in conjunction with a sufficiently large forcing velocity, leads to a turbulent flow. We may hence conclude that the typical magnitude of the pressure forcing that drives atmospheric flows, together with the presence of the Earth's surface leads to the fact that the atmospheric boundary layer is characterised through turbulence. Sometimes, the height where the turbulence 'vanishes' is even used to define the boundary layer height. Clearly, turbulence may be encountered also in other parts of the atmosphere. For example, the flow in a cumulus cloud is also highly turbulent. Similarly, so-called clear air turbulence can occur throughout the troposphere (often due to breaking waves) and sometimes leads to unpleasant experiences in airplanes. However, far away from the surface (i.e. from a permanent 'source' of turbulence) it is quickly dissipated and only occurs as intermittent phenomenon.

Tab. 2.2: Exchange properties of laminar and turbulent flows. In this table 'L' is used as an generic lengths scale while 'R' stands for the radius of a pipe.

	Laminar	Turbulent
Mixing length	**~ Free path length** [$\mathcal{O}(10^{-8}$ m$)$] Dependent on density and temperature ⇒ statistical state of fluid	**~ Dimension of domain** (e.g. R of pipe, distance from surface in the atmosphere) Dependent on *flow*
Irregularities (without external forces)	**Microscopic** [L ~ λ] Balanced in ~ 10^{-9} s	**Macroscopic** (L ~ R) May 'survive' L/U = t, e.g.: L = 0.1 m U = 1 ms^{-1} t= 0.1 s L = 1000 m U = 5 ms^{-1} t=200 s
Efficiency of exchange	**Molecular viscosity** [$\mathcal{O}(\sim 10^{-5} m^2 s^{-1})$] Dependent on microscopic kinetics (λ, T) ⇒ *property of the fluid*	**Eddy-Viscosity** ~ 10^6 ν Dependent on *macroscopic flow properties* ⇒ for the most part *not a property of the fluid*
Reynolds Number	**<2000**	**» 2000**

References Chapter 2

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BOX 'Tools to describe atmospheric flows'

In boundary layer meteorology, often certain definitions and assumptions are used to simplify the equations. These are

1) *Co-ordinate system*: Usually a Cartesian system is employed, with the x-axis pointing in the direction of the mean wind, the y-axis is the *lateral* direction and the z-axis points vertically up. The corresponding velocity components are denoted u,v,w. Alternatively, the coordinates may be denoted x_1,x_2,x_3 and the components of the wind vector become u_1,u_2,u_3.
2) *Einstein summation notation*: A) If a subscript occurs twice in a term of an equation it means that actually a summation over the three spatial

directions is meant. Thus $\partial u_i / \partial x_i$ means $\partial u_1 / \partial x_1 + \partial u_2 / \partial x_2 + \partial u_3 / \partial x_3$. B) On the other hand, if an index is not summed in one of the terms in an equation, it must not be summed in any other term of the same equation. This then means that actually three equations are referred to (with the index taking on 1, 2 *and* 3, respectively).

3) *Incompressibility:* Atmospheric flows are to a high degree incompressible, which means that a flow convergence or divergence in one direction is compensated by a change in the velocity in another direction rather than a change in density. Thus the mass conservation equation becomes $\partial u_i(\vec{x},t) / \partial x_i = 0$.

4) *Newtonian fluid:* A Newtonian fluid is defined as a fluid for which the shear stress tensor, σ_{ij} is proportional to the deformation rate in the fluid: $\sigma_{ij} = \rho \nu s_{ij} =: \rho \nu \left(\partial u_i / \partial x_j + \partial u_j / \partial x_i \right)$. Here, ρ is the density and ν the so-called kinematic viscosity.